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THE USE OF CHAFF IN SPACE
AS A JAMMING DEVICE BETWEEN
GROUND STATIONS AND SATELLITES

THESIS

Alan R. Sterns
Captain, USAF

AFIT/GSO/ENP/88D-6

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DEPARTMENT OF THE AIR FORCE

AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Space Operations Engineering

Alan R. Sterns, B.S.

Captain, USAF

December 1988

Approved for public release; distribution unlimited

Preface

The purpose of this effort was to predict the time evolution of a chaff cloud deployed in orbit around the earth. The idea of using a space-deployed chaff cloud as a jamming device for satellite communications was proposed by Captain Tommy Brown in 1987.

The bulk of the research was in two areas: dissecting and understanding the statistical mechanics approach to satellite breakups as presented by William Heard of the Naval Research Laboratory, and reassembling the information into a useable form in spreadsheet programs so the chaff cloud could be studied at various points in time and for various dispensing velocities. The work is far from complete, however; and further research into this area would be useful. Of particular value would be a computer program which could more efficiently model the chaff cloud over time, thus saving countless hours for any further researcher.

Throughout the course of my research, I have had extensive help from my thesis advisor, Lt Col Howard E. Evans II and will be forever in his debt. I would also be remiss if I did not mention th. close friends who were always there to listen to my tirades and helped keep me

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Abstract

This stuoy predicts the time evolution of the
attenuation characteristics of a chaff cloud deployed in
orbit around the earth. The study consists of three parts:
applying the statistical mechanics solution of a satellite
breakup model by William Heard of the Naval Research
Laboratory, solving for particle density at any time after
dispensing, and calculating the attenuation of an 8 GHz
signal through the cloud. The study shows that significant
levels of signal attenuation can be achieved, with
attenuations of greater than -50 dB lasting for several
hours.

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THE USE OF CHAFF IN SPACE AS A JAMMING DEVICE
BETWEEN GROUND STATIONS AND SATELLITES

I. Introduction

This chapter provides a general background for the
problem of deploying a chaff cloud in orbit about the earth
with the purpose of jamming satellite communications. It
describes why this research effort was needed, and exactly
what the problem entailed. Finally, it provides an outline
of the scope, assumptions and sequence of presentation which
follow.

Background

To fully understand the need for and scope of this
research effort, some background knowledge concerning both
chaff and statistical mechanics is required. This section
provides the basic information needed to understand the
orbital chaff cloud jamming problem.

<u>Chaff.</u> Reflective chaff is an electronic
countermeasure (ECM) device consisting of a large number of
tiny dipoles that are deployed in a rapidly expanding
"cloud," and is used to attenuate incident electromagnetic
signals. The individual chaff particles affect this

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attenuation by reflecting and absorbing portions of the incoming signal (Brown, 1987:4-1). The wavelengths of the incident signal most attenuated by a chaff cloud are those which are approximately double the length of the individual chaff particles (Butters, 1982:197). Some a priori knowledge of the incident signal is thus required so that the chaff particles can be constructed at the proper length for maximum signal attenuation to occur.

Chaff is routinely deployed in the atmosphere by the Air Force and Navy as a decoy to intercept radar signals targeted for planes and ships, thus allowing the intended targets to avoid detection and subsequent destruction.

Due to the aerodynamic forces which act on the individual chaff particles in the atmosphere, a chaff cloud rapidly blooms (expands) and begins falling to earth shortly after deployment. Therefore, it is a very transient ECM device. In space the aerodynamic forces are minimal, however, so a chaff cloud would be a relatively long-lived entity.

Brown has proposed that chaff in space could be effectively used to jam satellite communications if a cloud of chaff with sufficient density and of the proper resonant wavelength were used (Brown, 1987:4-1 to 4-24). In order to assess the utility of deploying chaff in space, a model

describing the physical blooming characteristics of the chaff cloud needs to be developed.

A blooming model for chaff in outer space would have to include the following information: the size of the cloud in three dimensions at any given time, the particle density of the cloud at any given point in space, and the rate of expansion of the cloud in all directions. The cloud model would also have to be useable from any reference point on the earth to determine whether a signal sent between the ground and a satellite, or vice versa, would be properly attenuated.

Statistical Mechanics Approach. Ideally, the position, velocity, and density of a collection of particles can be described using Newton's Laws of Motion, particularly his Third Law: $\underline{F} = m\underline{a}$, where $\underline{F}$ is the force acting on a given particle, m is the particle's mass, and $\underline{a}$ is the acceleration vector of the particle in three dimensions. When the number of particles whose motion needs to be described becomes large, however, the use of Newton's Third Law becomes impractical and an approach using statistical mechanics is often taken.

Statistical mechanics refers to the study of the collective properties of a system consisting of a large number of small particles whose exact position and velocity

components are either unknown or difficult to describe (Johnson and McKnight, 1987:56). Since the chaff cloud problem involves billions of individual particles, a statistical mechanics approach is justified.

Problem Statement

This study uses the statistical mechanics approach described by William B. Heard of the Naval Research Laboratory in his paper "Dispersion of Ensembles of Non-Interacting Particles" and describes the time history of the size, shape, and particle density of a space deployed chaff cloud based on stated initial conditions at an ideal spherical deployment. The study then uses the size and shape information for the cloud and the particle densities within the cloud to determine the attenuation of a signal from a ground-based antenna to a satellite located behind the cloud. The attenuation study examines signal attenuation along a column which runs from the part of the cloud closest to the earth, through the center of the cloud, and out the part of the cloud nearest the satellite. Since the cloud is continually changing in size, shape and density, the attenuation study is accomplished by taking "snapshots" of the cloud at various times after initial deployment. The dispensing velocity of the cloud is also varied to determine the dispensing velocity which provides

the most effective attenuation over a specified period of time. This information is then used to determine if jamming occurs, and if so, over what time period and for what dispensing velocities.

Scope of Research

The research effort examines the following:

1) existing data which describes the time evolution of a collection of particles in earth orbit;

2) the field of statistical mechanics as it applies to the chaff cloud problem;

3) Heard's solution to the statistical mechanics problem of numerous particles in orbit about a large body, and its application to the chaff cloud problem;

4) Brown's stated requirements for the attenuation of electromagnetic signals through a chaff cloud and equations appropriate for signal attenuation calculations.

Evolution of Particles. A knowledge of the anticipated time history evolution of a chaff cloud is essential in determining if the results obtained via statistical mechanics provides accurate data. Several research efforts have addressed this area in the past. The results of two of them have been examined in this study to provide a basis with which to compare the results of the statistical mechanics solutions.

The first of these studies is the work done by Ross in 1961 which provided a theoretical basis for the evolution of a group of particles deployed from a satellite in earth orbit (Ross, 1961:79-83).

A second research effort entitled "Project Westford" examined the usefulness of a dipole cloud in earth orbit as an aid to long-distance, point-to-point communication. Project Westford got into full swing upon the release of 480 million dipoles into earth orbit on 10 May 1963. The resulting cloud of particles was used to reflect radio signals between a ground-based transmitter and a receiving antenna located at a distant location (Overhage and Radford, 1963:452).

The long-term (days to months) evolution of the chaff cloud, although an interesting aspect of the problem, is not addressed in-depth. Since the current problem involves only whether or not a signal can be attenuated, and if so, for how long, the final shape of the cloud is of no particular interest. After the attenuation of the incident signal drops below a specified value, the evolution of the cloud is no longer pertinent to this study.

Statistical Mechanics Solutions. Statistical mechanics has applications in many wide ranging disciplines. Therefore, a list of some of these applications are provided

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to further introduce the subject. First, specific areas of
application of statistical mechanics are listed. Then, the
Boltzmann equation, which is used to describe the time
history of the distribution of a collection of particles, is
presented.

Orbital Statistical Mechanics. Several researchers
have done work using statistical mechanics which they claim
can be used to describe such systems as the rings of saturn,
satellite breakups, the asteroid belt, meteor streams, and
moving star clusters (Hameen-Antilla, 1976:145; Heard,
1976:63; Johnson and McKnight, 1987:56-57). The results of
these efforts, when compared with the results of the two
particle evolution studies mentioned previously are used to
solve the problem at hand. In particular, the local
densities at various points throughout the chaff cloud are
determined. This in turn leads to the actual number of
particles lying within various subvolumes within the cloud.
These particle densities are also examined at various time
intervals after deployment of the cloud. The attenuation of
the signal through each subvolume (and subsequently the
entire cloud) is then determined.

Signal Attenuation. In his 1987 Master's thesis, Brown
provided several equations describing the attenuation of a
radio signal through a chaff cloud. His procedure depends

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on the number of particles encountered by the incident signal. Brown also described the size of a chaff cloud required to jam an area the size of Washington D.C, the required altitude of the chaff cloud for this scenario, and time requirements for signal attenuation (Brown, 1987:4-3 to 4-5, 4-13 to 4-17). In order to prevent the unnecessary duplication of effort, his results are applied directly to the above statistical mechanics solutions to determine the attenuation of the signal through each of the subvolumes described above. By examining the attenuation through subvolumes which line up directly with each other along a central column through the chaff cloud, and by multiplying together the percentage of the signal which passes through each of these subvolumes, the percentage of the signal which penetrates through the cloud is determined.

Assumptions

Several general assumptions are made to help simplify the calculations and keep the scope of this research effort manageable. The assumptions are as follows:

1) All the chaff particles can be instantaneously and ideally deployed in orbit at the specified time, t_0.

2) The cloud will follow an ideal, circular orbit around a non-rotating earth and will experience no perturbations.

3) There will be no physical interaction between individual chaff particles after deployment.

4) The initial dispersion pattern will be spherical.

5) The electromagnetic signal will always be directed normally to and through the point of initial dispersion (centroid) of the cloud to a geosynchronous satellite.

6) The incremental velocity imparted to the chaff particles upon deployment is very much less than the velocity of the parent body in its orbit.

Sequence of Presentation

First, the research is presented beginning with a review of current literature relating to the problem of deploying a chaff cloud in earth orbit. As outlined above, this includes a look at the evolution of the cloud using classical approaches, a discussion of statistical mechanics, a presentation of specific statistical mechanics solutions to orbital problems, and a synopsis of signal attenuation as described by Brown.

A short methodology section briefly describing the process involved in the solution of the problem is next, followed by a section building on the theories outlined in the literature review section. Appropriate adaptations for the chaff cloud problem are included.

Following that is a section that provides the solutions to the chaff cloud problem, including two-dimensional

graphical representations showing the physical dimensions of the chaff cloud as a function of time. These results are then analyzed using the attenuation model to show the time period over which effective jamming by the chaff cloud occurs.

The final section provides some conclusions regarding the chaff cloud problem and recommends some areas of study where future research might be useful.

II. Review_of_Literature_and_Background_Development

This chapter provides an in-depth review and some development of the background information which forms the basis for this research effort. It is divided into the following sections: discussions of chaff cloud evolution in general, a review of statistical mechanics, solutions to orbital problems similar to the chaff cloud problem using statistical mechanics approaches, and a review of the theory of signal attenuation through a dipole cloud.

Cloud_Evolution

As mentioned in the previous chapter, several researchers have addressed the problem of the evolution of a collection of particles that has been released from a parent satellite in earth orbit. Since this paper is mainly concerned with the density of the chaff cloud and an integration across its volume, and since statistical mechanics will be used to provide the actual numbers to be used, this section does not apply any rigorous mathematics. Instead, only a description of the most general aspects of chaff cloud evolution in earth orbit will be discussed.

Orbital_Mechanics_Review. In order to fully understand a satellite's orbital motion about the earth, a knowledge of

the basic orbital parameters is required. This section provides a review of some of the basic concepts and definitions associated with orbital mechanics.

The coordinate system for an earth-orbit is a right-handed, inertial system with the origin at the center of the earth. The I-axis points in the Vernal Equinox direction and passes through the earth's equator. The J-axis is located 90 degrees from the I-axis along the equatorial plane. The K-axis is orthogonal to the other two principle axes and points through the north pole (Bate and others, 1971:58).

There are six basic orbital parameters: semi-major axis (a), eccentricity (e), inclination (i), longitude of the ascending node (Ω), argument of perigee (ω), and time of perigee passage (T). All of the orbital parameters are described in "Fundamentals of Astrodynamics" by Bate, Mueller, and White (Bate and others, 1971:58). The orbital parameters are represented pictorially in Figure 1.

Three additional aspects of a satellite's motion which are also important to the chaff cloud problem, although not included among the basic orbital parameters, are the period of the satellite's orbit (P), the specific mechanical energy of the orbit (E), and the satellite's mean motion (n).

The period is the amount of time it takes a satellite to complete one revolution about the earth and is usually

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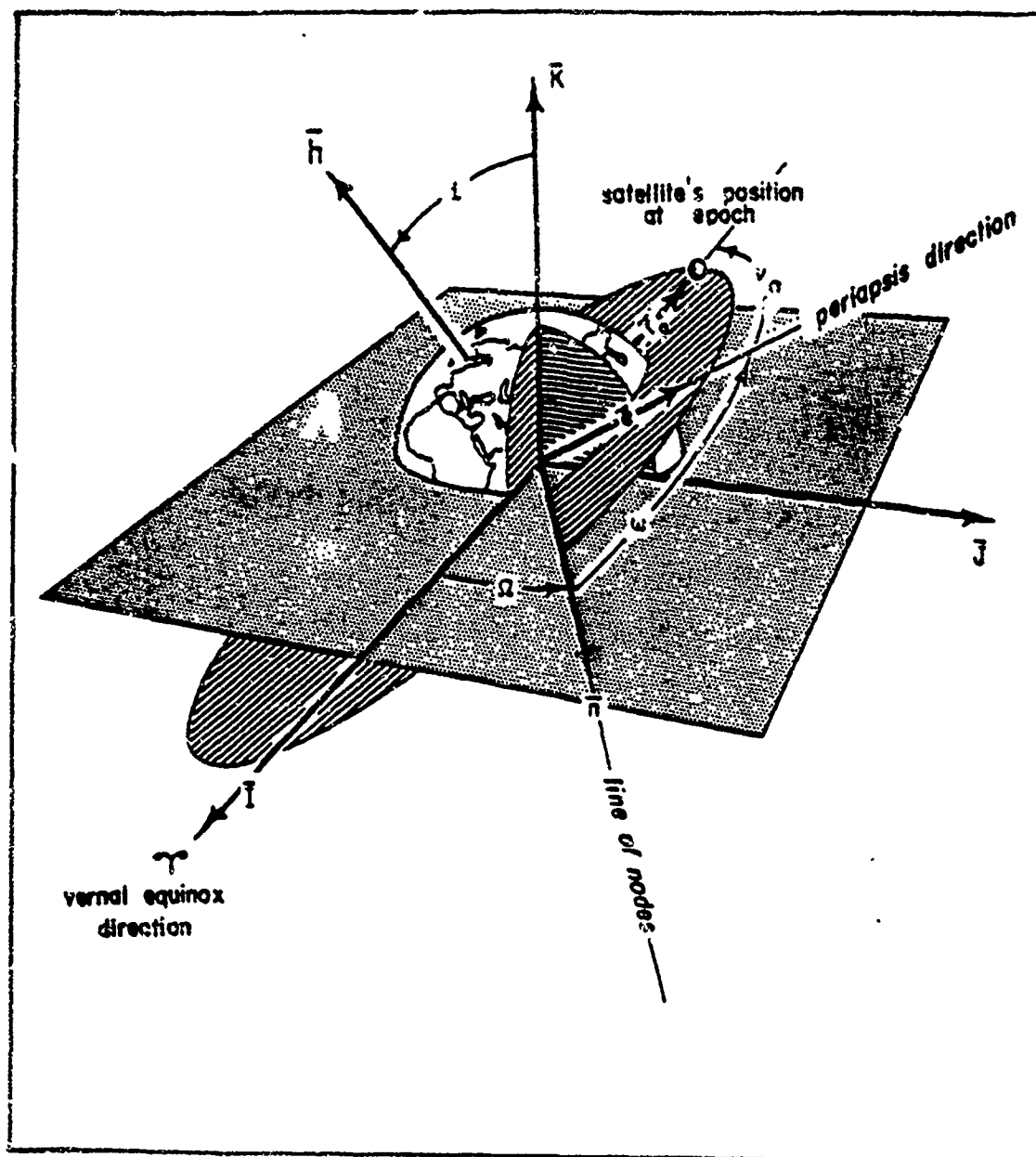
Figure 1. Basic Orbital Parameters
(Bate, 1971:59)

measured in minutes. A satellite's period is defined as follows:

$$P = 2\pi(a^3/K^{1/2})\qquad(1)$$

where a is the semi-major axis and K equals the Universal Gravitational Constant (G) multiplied by the mass of the earth (M) (Bate and others, 1971:33).

The specific mechanical energy of each particular orbit is constant and related to K in the following manner:

$$E = v^2/2 - K/r\qquad(2)$$

Here, v is the magnitude of the satellite's velocity and r is the distance from the center of the earth to the satellite (Bate and others, 1971:16).

Mean motion is the time it takes a satellite to complete one radian of travel about its orbit and is defined as follows:

$$n = (K/a^3)^{1/2}\qquad(3)$$

where K and a are the same as in Eq (1) (Bate and others, 1971:185).

From the relationships described above, it can be shown that any change in a satellite's energy directly affects both the satellite's period and the semi-major axis of the satellite's orbit (Thomson, 1986:58). Shapiro states that a

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change in satellite velocity will not change the energy of
the satellite's orbit unless a component of the velocity is
in the direction of satellite motion. Velocity components
in the other two orthogonal directions have effects only on
the other orbital parameters, particularly 'e' and 'i'
(Shapiro and others, 1964:483).

   Preliminary_Assumptions_and_Setup. To understand a
description of the evolution of a particle cloud in orbit,
it is best to begin by looking at simplistic examples and
expand the complexity one step at a time. Therefore, it
will be assumed that the parent satellite (the satellite
which dispenses the chaff particles) starts in a perfectly
circular ('e' = 0), equatorial ('i' = 0) orbit about the
earth. At this point, the specific values for the other
orbital parameters and the values for 'P', 'E', and 'n' are
unimportant. As noted in chapter I, any perturbations to
the orbit caused by anything other than the velocity changes
noted below are assumed to be negligible.

   For these examples, the shape of the parent satellite
will be assumed to be cylindrical with the satellite
spinning about its longitudinal axis. For simplicity, the
cloud will be assumed to consist of small particles
initially located inside the spinning parent satellite.
Dispensing occurs when the outer shell of the parent

satellite is instantaneously removed. This allows the particles, no longer held in place by the shell but still moving at the angular velocity imparted by the spinning of the parent satellite, to spread radially from the parent.

The rest of this section will examine four types of particle dispensing from the cylindrical particle satellite as described by Ross in his paper "The Orbital Motion of Pellet Clouds" and Shapiro, Jones, and Perkins in their paper "Orbital Properties of the West Ford Dipole Belt" (Ross, 1961:79-83; Shapiro and others, 1964:482-492). Except where noted, all the material for Cases I and II comes from Ross's paper; Cases III and IV are entirely from the Shapiro paper.

The first two cases assume instantaneous dispensing of the particles, and the parent satellite's orientation is as shown in Figure 2. The case where the spin axis is oriented axially in relation to the earth will not be specifically discussed, but will be automatically included in Cases III and IV as discussed below.

Cases III and IV involve dispensing the particles over a period of time and are also shown in Figure 2. The release of particles over time could be accomplished by releasing only a fraction of the outermost particles within the parent at any one instant in time. In these cases, the direction of spin axis orientation is assumed to be time

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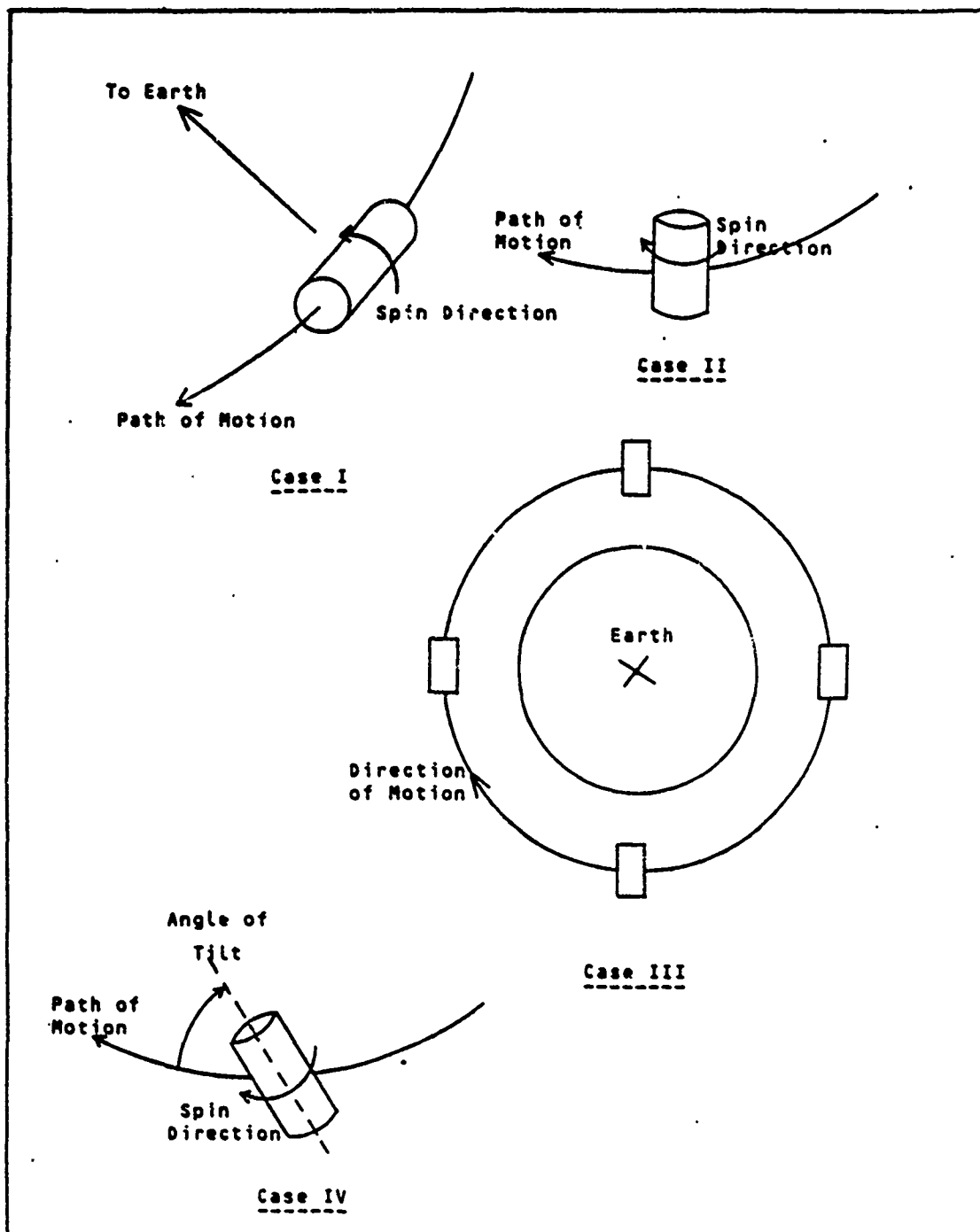
Figure 2. Satellite Orientation for

Cases I, II, III, and IV

independent and, therefore, the spin axis will continually change its orientation angle with respect to the path of satellite motion as the satellite moves around the earth. The resultant dispensing of particles over time causes spreading in all directions relative to the parent satellite.

Case I: Spin Axis Along Satellite's Path of Motion. In this first case, Ross says that since no incremental velocity is imparted upon the particles in the direction of satellite motion, no energy change is imparted to their respective orbits. Therefore, no changes in their orbital periods occur.

The velocity changes which occur happen only perpendicular to the parent satellite's path with the subsequent effect that both the eccentricity and inclination change. The eccentricity change is caused by the radial component of the particle velocity with respect to the orbital plane of motion, while the inclination change is a result of the velocity component perpendicular to the orbit plane.

Another interesting effect is that since a change in period does not occur, all particles must return to their original locations in the parent orbit at the end of each subsequent revolution about the earth. This means that

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although the incremental velocity imparted to the particles causes an expansion of the cloud, the expansion reaches a maximum halfway around the orbit. At this point, the cloud begins to contract and will return to its original size and shape at the original point of dispensing. The combination of the change in values for 'e' and 'i' has the additional effect of causing the "ends" of the cylindrical cloud to tilt sunusoidally with time. Figure 3 illustrates the contracting/expanding/tilting particle cloud over time. In the figure, 'nT' is the portion of the orbit (in radians) the cloud has traversed, the 'S' direction is in the direction of satellite motion, the 'R' direction is pointed radially to the earth, and the 'W' direction is out of the orbital plane.

The effect of the angular velocity of the parent satellite's spin is important to the dispensing in that the outermost particles undergo the biggest changes in velocity (and thus 'e' and 'i') and the innermost particles experience the smallest changes. Because of these variances in the delta velocities, a smear of particles develops between the parent orbit and the orbits of the particles with the maximum eccentricity and inclination changes.

It is important to note that since the velocity changes occur instantaneously and no further accelerations or velocity changes are involved, the changes in 'e' and 'i'

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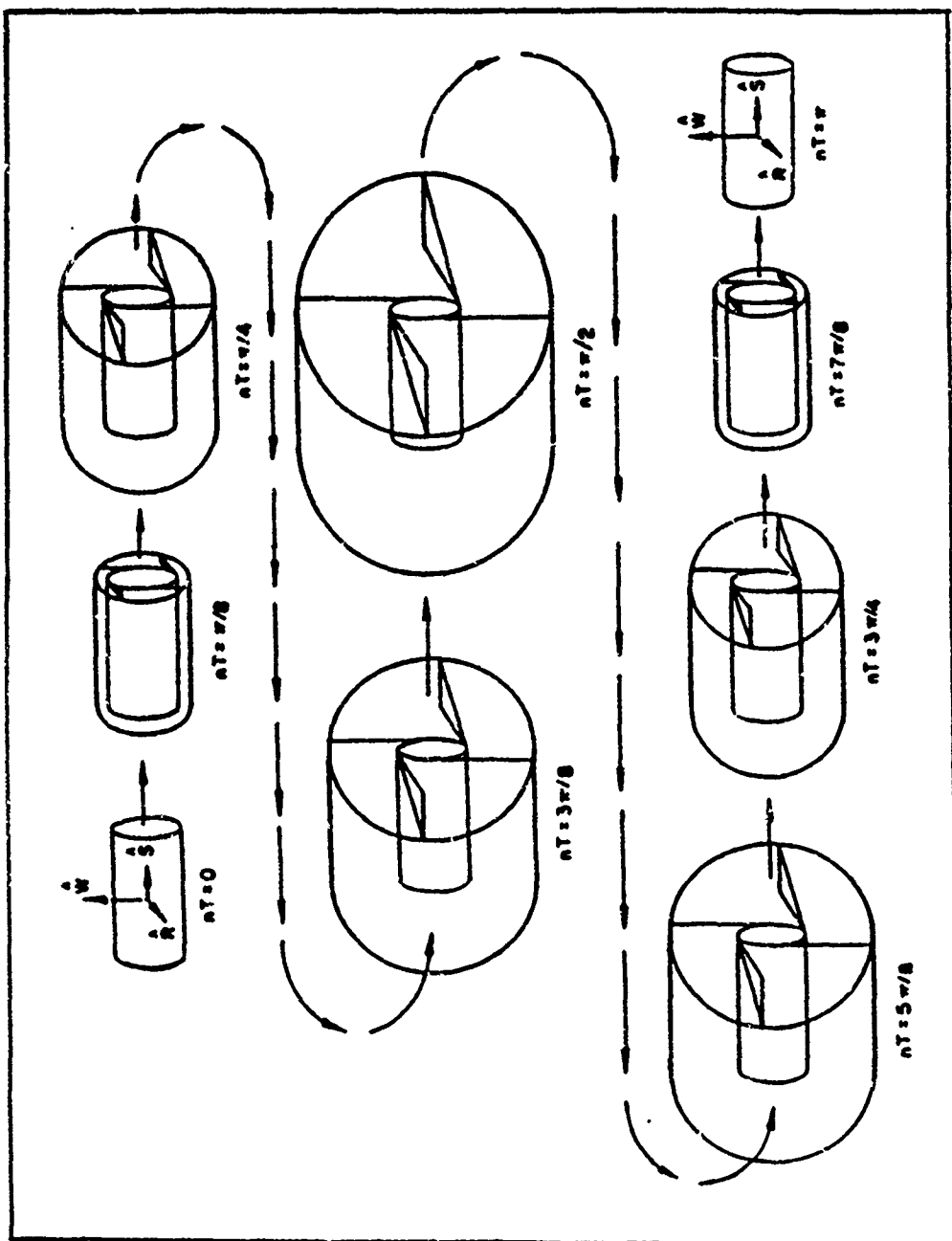
Figure 3. Chaff Cloud Evolution for Case I
(Ross, 1961:82)

have an upper limit. This limit is what causes the cloud to reach its maximum size at the halfway point in the orbit.

Case II: Spin Axis Perpendicular to Orbit Plane. With the spin axis perpendicular to the orbit, there is one component of velocity directed radially from the earth in the orbital plane and one component in the direction of satellite motion. Since no velocity component can cause an inclination change, the spread of the particles out of the orbit plane is solely determined by the initial length of the parent satellite.

The velocity component in the radial direction has the same effect as it had in the previous case, causing a change in the eccentricity of the satellite's orbit. However, the velocity component which acts in the direction of satellite motion introduces a new effect into the problem. By altering the particle velocity in the direction of motion, the energy of the orbit is changed. This alteration in turn causes a change in 'P' and 'a' of the orbit.

The effect of the differences of the angular velocities of the particles, as was noted in Case I, is that the outermost particles undergo larger changes in velocity (and thus 'P' and 'a') than the innermost particles. This causes either an increase or decrease in orbital period depending on the direction of the velocity vector in relation to the

satellite's motion. Again, a smear of particles develops between the parent orbit and the highest and lowest orbits caused by the period changes.

Since the period changes are permanent if no other perturbations are considered, the particles continue to spread forward and backward along the orbit (with respect to the parent satellite) at rates equal to their respective changes in period from the initial orbit. For example, if a period change of one minute is imparted upon a particle, it will move away from the parent satellite at a rate of one minute per orbit. Eventually, the cloud expands to the point that the forward moving particles meet the backward moving particles halfway around the orbit from the parent satellite. Since the expansion due to period changes of the particles continues indefinitely, a toroid ultimately develops around the earth.

The result of the eccentricity and period changes acting together to change the orbit (for those particles that have components of velocity in both the radial direction with respect to the parent satellite and the direction of satellite motion) are different than for Case I because of the period changes involved. The eccentricity causes some oscillations of the height of the apogee and perigee points with a maximum occuring halfway through the first orbit. However, due to the fact that virtually every

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particle has at least a small component of velocity along
the direction of motion, the overall tendency of the cloud
is not to continue oscillating each orbit. Instead, the
cloud elongates along the orbit in a manner similar to that
shown in Figure 4.

Case III: Long-Term Dispensing with Spin Axis in
Orbital Plane. With the spin axis lying in the orbital
plane, its orientation is the same as that in Case I exactly
twice each orbit. These points are 180 degrees from each
other in the orbit. At a point 90 degrees from either of
these points, the spin axis is oriented radially with
respect to the earth. At any other point in the orbit, the
spin axis is at some other orientation.

With the spin axis oriented precisely perpendicular to
the path of motion for only an instant of time twice each
orbit, only a fraction of the particles receive velocity
components exactly in the direction of motion. Therefore,
the number of particles with the greatest period change is
significantly less than in the previous two cases. However,
since the particles are dispensed in all directions, all
particles receive some change in period (except in those
instances with the spin axis exactly parallel to the
satellite's motion), unlike Cases I and II.
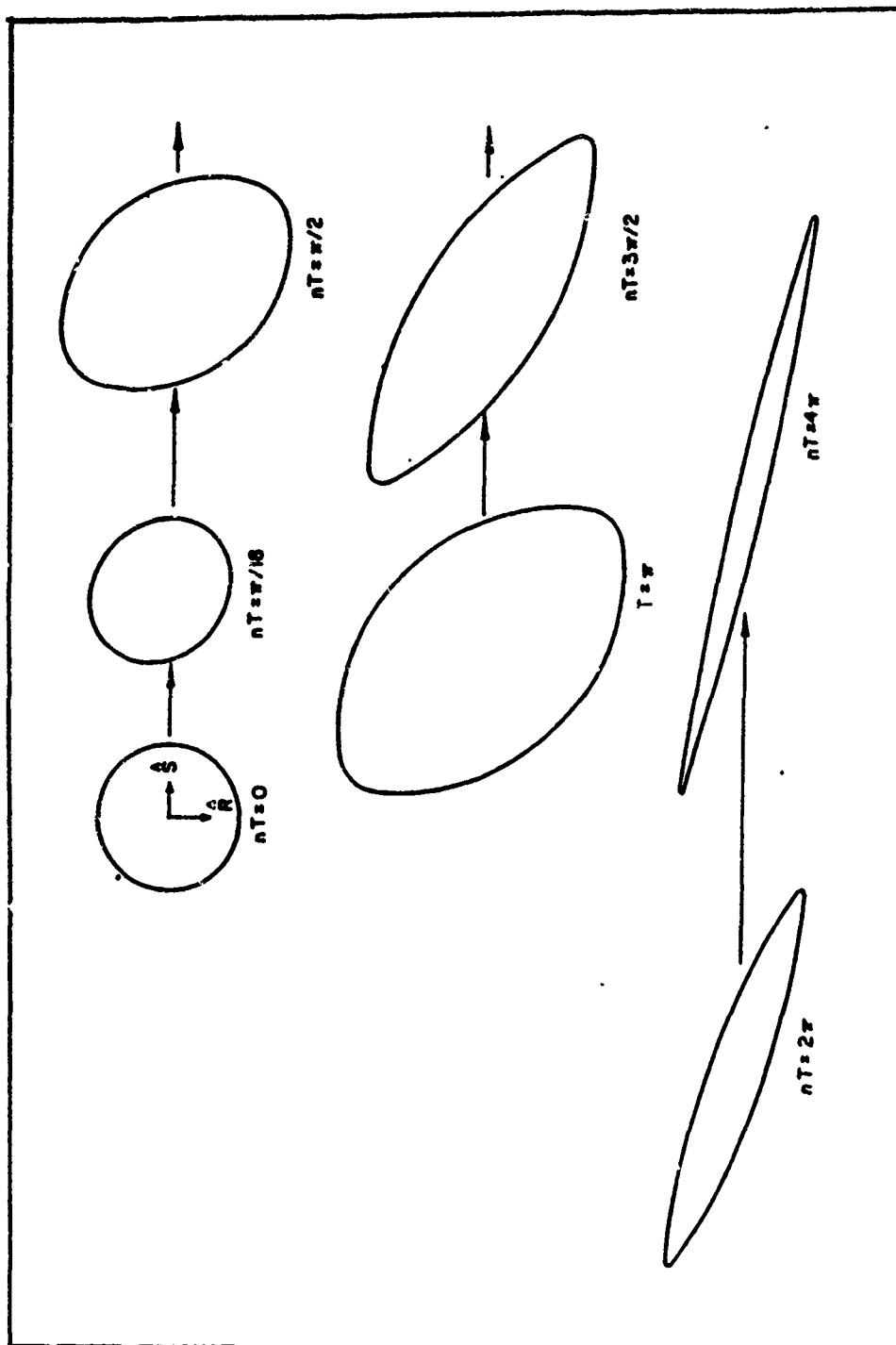
Figure 4. Chaff Cloud Evolution for Case II
(Ross, 1961:82)

With a continually changing orientation of the spin axis relative to the satellite's motion, the velocity components influencing eccentricity and inclination change also. The maximum changes in eccentricity and inclination occur with the spin axis oriented along the path of motion.

When effects of the various changes in period, eccentricity, and inclination are combined with the effects of different velocity magnitudes due to the shrinking size of the dispenser as time progresses, a distribution of particles occurs that has a peak density near the parent orbit. The density rapidly diminishes farther out from the parent satellite.

Shapiro notes an interesting result when the overall cloud is described pictorially. Since the largest change in period occurs only for those particles released when the spin axis is oriented radially from the earth, particles released here will move ahead or lag behind the parent satellite the farthest. Those that lead are in orbits with lower perigee altitudes, and those that lag are in orbits with higher apogee altitudes than the parent satellite.

Assume the spin axis is aligned radially with respect to the earth at u' = 0 degrees and u'' = 180 degrees as shown in Figure 5. For those particles released at u', the following situation develops: all particle orbits intersect the parent orbit at this point, with the lagging particles
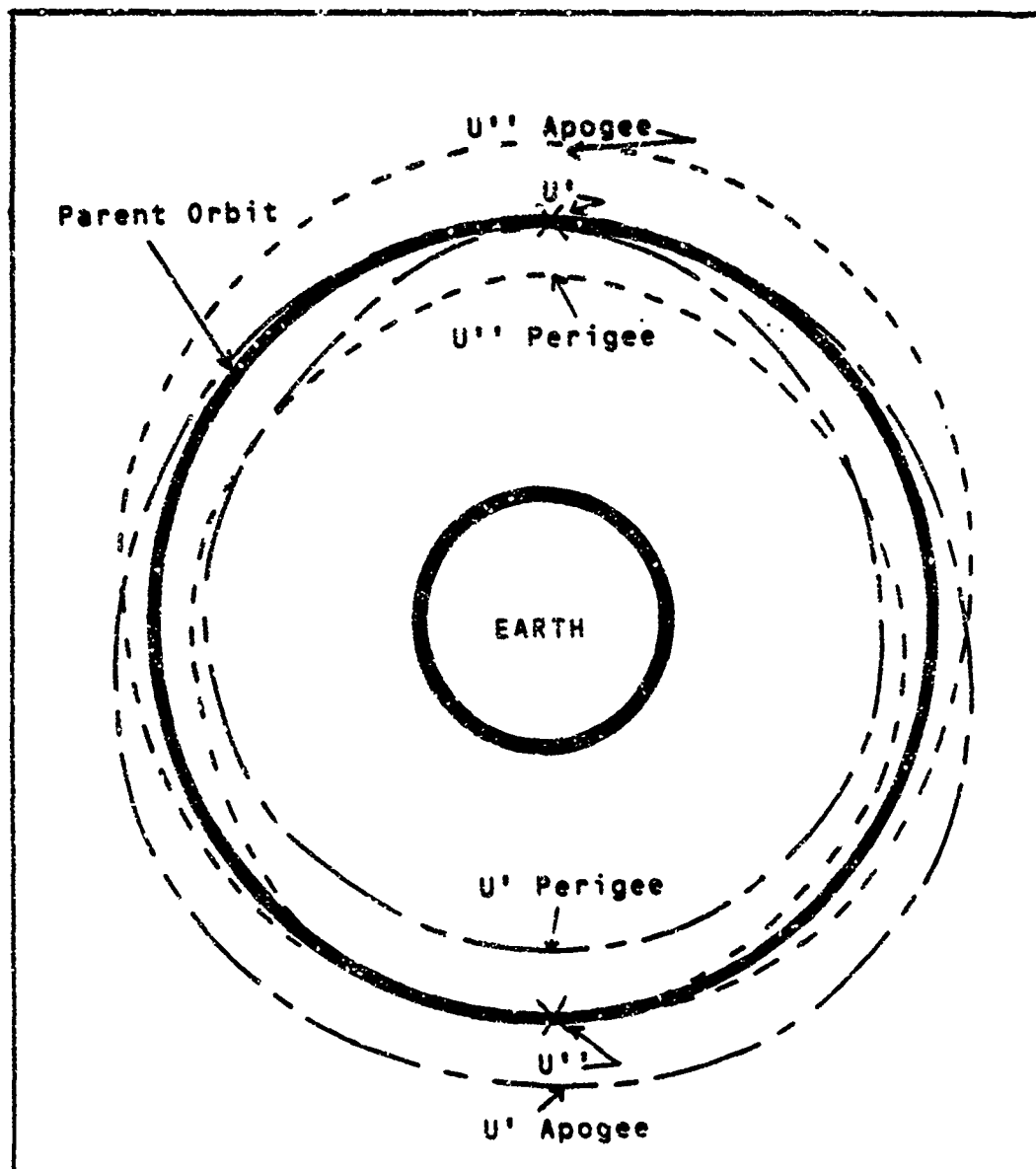
Figure 5. Dispensing Setup for Case III

having perigees and the leading particles having apogees here. For those particles released at u'', the same thing happens only it is one-half orbit removed. At the u'' point, the particles released at u' are all at either perigee or apogee of their respective orbits and are ahead or behind of the parent satellite, respectively. A similar situation occurs for the particles released at u'' when they reach the u' point in the orbit.

The interesting result is that at both u' and u'', there are particles that were released at the opposite end of the orbit, some of which are at perigee and some of which are at apogee. Therefore, at u' there are particles leading the parent satellite in a slightly lower orbit (u'' particles at perigee), particles lagging behind the parent satellite in a slightly higher orbit (u'' particles at apogee), and particles leading and lagging the parent satellite at the same altitude (u' particles). A similar result applies to the u'' points. At these points, a four pointed cloud like the one shown in Figure 6 develops. At the points in the orbit where the spin axis is aligned with the direction of satellite motion (90 and 270 degrees), a smooth transition between particle orbits occurs.

According to Shapiro, the overall effect of the particle cloud "can thus be envisioned as a two-headed
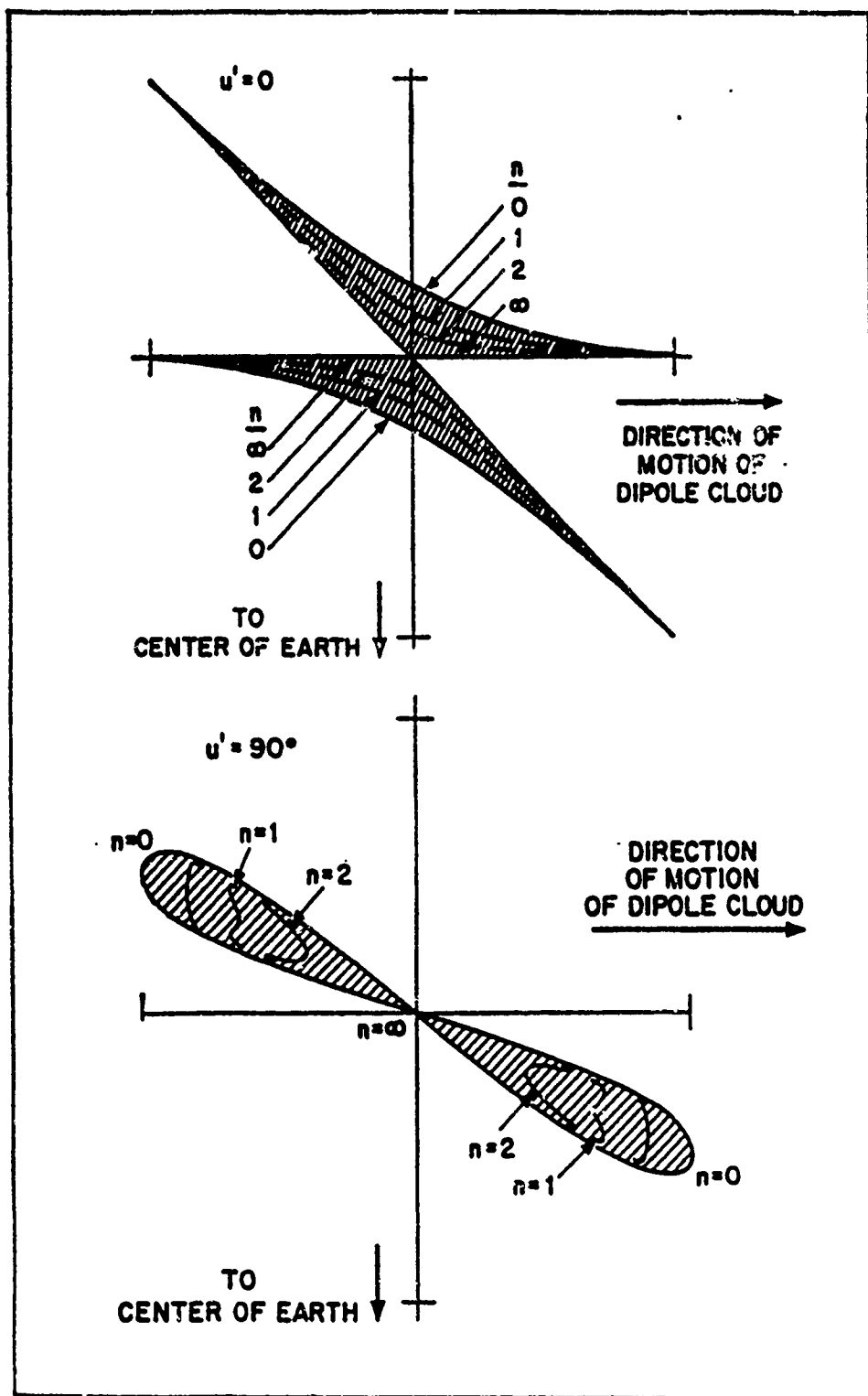
Figure 6. Chaff Cloud Evolution for Case III
(Shapiro, 1964:491)

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dragon, snapping its jaws open and closed as it circulates around the earth" (Shapiro and others, 1964:490).

Case IV: Long-Term Dispensing with Inclined Spin Axis. The only differences between this case and Case II above are that the angle between the spin axis and the orbital plane can be set anywhere between 0 and 90 degrees (instead of being placed rigidly at 90 degrees), and that the dispensing takes place over a long period of time. Shapiro makes some mention of the situation in which the angle is 90 degrees, but the results agree with Case II above, so it will nc be discussed here.

For Case III, Shapiro says that the greatest inclination change for the particles occurs at the point along the orbit where the largest velocity component perpendicular to the orbital plane exists. He also says that the greater the angle between the dispenser and the orbital plane, the smaller the spread in the distribution of the inclinations of the particle orbits. This is because at 90 degrees (Case II) no inclination changes occur, and at 0 degrees (Case II') virtually all particles ex ibit some inclination changes. Because of these effects, the inclination changes undergone by particles can be kept to within a desired range by changing the angle of tilt of the dispenser relative to the orbital plane.

The effects of inclining the spin axis relative to the orbital plane on the other orbital elements is a combination of all the effects previously mentioned. At an inclined angle of 0 degrees, the distribution of the number of particles with various period changes is the most pronounced, since relatively few particles experience the greatest possible period change. For greater angles of inclination, larger numbers of particles can undergo changes in period, with a maximum number occuring at an inclined angle of 90 degrees (Case II). Thus, Shapiro points out, although the maximum period change can be experienced by particles at any inclined angle (assuming their velocity component is aligned with the direction of motion), the distribution of the number of particles with any given period change is very angle dependent.

Finally, Shapiro says that no matter what the angle of tilt, the resulting distribution of particles is centered about the parent satellite's orbit, with the largest concentration of particles in the vicinity of the parent orbit. The resulting cloud for this case also resembles Figure 6 except for slight differences in both the distribution of the particles and spreads in the inclinations of the particle orbits.

Statistical Mechanics

As mentioned previously, the motivation behind using statistical mechanics to study a collection of particles is that Newton's laws of motion become cumbersome when applied to a large number of particles. Statistical mechanics solves this problem by looking at the collection of particles as a single entity and analyzing its behavior over time.

Several areas of physics routinely utilize statistical mechanics to aid in the solution of problems. Among them are plasma physics and kinetic gas theory. However, since none of these disciplines directly apply to the chaff cloud problem, no further mention of them will be made.

To properly examine the behavior of the particles over time as a group, a distribution function is required. A distribution function describes the location, size and density of the collection of particles as a function of time. Once the distribution function is known and appropriate initial conditions are applied, the time history of the collection of particles can be determined.

An equation commonly used to describe the time evolution of distribution functions is the Boltzmann equation which, according to Chen, is as follows:

$$df/dt + \underline{v} \cdot \nabla f + \underline{F}/m \cdot df/dv = (df/dt)_{coll} \qquad (4)$$

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Here f represents the function $f(\underline{r},\underline{v},t)$ or the number of particles per cm^3 as a function of position ($\underline{r}$), velocity ($\underline{v}$), and time (t). df/dt is the derivative of f with respect to time. ∇f represents the gradient in (x,y,z) space so that

$$\underline{v} \cdot \nabla f = \frac{df}{dx}\frac{dx}{dt} + \frac{df}{dy}\frac{dy}{dt} + \frac{df}{dz}\frac{dz}{dt} \qquad (5)$$

$\underline{F}$ is the force acting on the particles, m is the mass of the particles, and df/d$\underline{v}$ is the gradient in velocity space so that

$$\underline{F}/m \cdot df/d\underline{v} = \frac{df}{dv_x}\frac{dv_x}{dt} + \frac{df}{dv_y}\frac{dv_y}{dt} + \frac{df}{dv_z}\frac{dv_z}{dt} \qquad (6)$$

The term on the right-hand side in Eq (4) describes the time rate of change of f due to collisions between particles (.en, 1977:206), and will be assumed to be negligible for the remainder of this work.

Orbital Mechanics Via Statistical Mechanics

This section examines the literature which specifically addresses the application of statistical mechanics to problems in orbital mechanics. All the examples addressed here have the common element of a large number of small satellites being released from a parent satellite. Applications of these examples to other problems in orbital

mechanics include examining the behavior of meteor streams
coming from a disintegrating comet, expanding star clusters,
asteroid belt fragmentations, and exploding earth satellites
(Heard, 1977:1025). Due to the similarities between the
case of an exploding earth satellite and the expanding chaff
cloud, the application of statistical mechanics to the chaff
cloud problem is fairly straightforward.

Applications. In their 1987 book Artificial Space
Debris, Johnson and McKnight discuss the usefulness of
applying statistical mechanics to determine the resulting
orbits of the pieces from a satellite breakup. They
describe the different particle orbits using an approach
outlined by Heard in a paper published in 1978. Heard's
work is described in the "General Theory" section below. In
addition to outlining Heard's approach to the problem,
Johnson and McKnight provide a useful summary of the
advantages of using statistical mechanics for analysis of
satellite breakups as opposed to using classical mechanics
approaches:

> First, the statistical approach requires less
> computational effort and does not require increased
> effort for larger breakups. It also provides a global
> definition to a problem which inherently does not lend
> itself to such an approach. (Johnson and McKnight,
> 1987:58)

Johnson and McKnight point out, however, that the use of statistical mechanics requires a conversion from basic orbital elements into analytic functions in order to solve the problems. They also mention that the nomenclature particular to statistical mechanics is generally not as well known as that of the classical approaches (Johnson and McKnight, 1987:58).

Heard has authored several articles on the topic of using statistical mechanics to model the propagation of a large number of particles in space over time. He states in his 1976 work that a collection of particles can be studied using either a classical approach or a continuum (statistical mechanics) approach. Like Johnson and McKnight, Heard also emphasizes the increased computational effort required by the former method. He goes on to say that since interactions among the particles are negligible, the solution via statistical mechanics requires nothing more than the application of the theory of linear, partial differential equations (Heard, 1976:63).

In the same paper, Heard also mentions the usefulness of a statistical mechanics approach in the field of astronomy to analyze different systems of particles that move in independent orbits about another body, but whose overall behavior tends to indicate a common origin, such as families of asteroids (Heard, 1976:63). In this paper, he

looks at what he calls the "direct" problem and the "inverse" problem. In the dir~·t problem, the final distribution of a collection of particles is examined beginning with the initial point of dispersion and ending at a future point in time. The inverse problem examines a collection of particles and attempts to recreate the moment of initial dispersion by moving backward in time (Heard, 1976:63-64). Since the current study is concerned only with the future spreading of a chaff cloud and not the origin of an existing chaff cloud, no further mention will be made of the inverse problem.

Heard's paper from 1977 examines the asymptotic (time increasing to infinity) behavior of a collection of particles (Heard, 1977:1025). Like the inverse problem, the final state of the chaff cloud is not important here, since the current study is concerned only with the cloud's spreading up to the point where the desired signal attenuation no longer exists. Therefore, except for the underlying statistical mechanics theory, Heard's 1977 effort is dropped from further discussion here also.

In 1978, Heard published a paper directly applicable to the problem of a satellite breakup which is useful for the current study. In this paper, he again examined the direct, inverse, and asymptotic problems; he also included an analysis of the "continuous source" problem. In the

continuous source problem, the dispersion of particles occurs over time. This is similar to Cases III and IV which were discussed above (Heard, 1978:1-2). Again, the direct problem will be the focus of this effort.

In 1976, Dasenbrock, Kaufman, and Heard co-authored a paper which examined the direct and inverse problems as they apply to satellite breakups and the subsequent debris cloud that develops. The specific stated purpose of this report was to

> Study the dynamical characteristics of an evolving fragment cloud in order to yield insight into both its future evolution for purposes of collision probabilities and to determine how the dynamics might be used to obtain the precise origin of the cloud. Specifically, satellite breakups are simulated to study the characteristics of their evolving fragment clouds. These characteristics are then used to determine how the time and place of a satellite explosion might be accurately determined once its fragment cloud has been observed. (Dasenbrock, 1976:2)

The paper, like the works discussed above, goes on to state that a statistical mechanics approach is called for to help ease the computational burden of following the orbital paths of the fragments (Dasenbrock, 1976:2).

Finally, Hameen-Antilla has also applied a statistical mechanics approach in his examination of a system of particles in which collisions between particles occur. In particular, the evolution of the rings of Saturn was discussed. The underlying purpose of the paper was to

improve on a previous work, also by Hameen-Antilla, in which statistical mechanics was used. The differences are that the second work allowed for differences in particle sizes and improved on the statistical orbit model used (Hameen-Antilla, 1976:145).

In each of Heard's three papers discussed above, he used the same basic theory, with slight variations in each work, to arrive at his final results. Aside from Heard's presentations, the only other direct applications to the chaff cloud expansion problem from the above list are the ones found in the Dasenbrock article and Johnson's and McKnight's book. However, the presentation of the theory in the Dasenbrock paper was identical to that in Heard's 1976 paper except for a few minor changes, while Johnson's and McKnight's presentation was taken l rgely from Heard's 1978 article. Therefore, Heard's method of presentation will be followed closely in the discussion below. Heard's 1978 paper used essentially the same format for presentation as his 1976 work, but in the .978 work some of the results were presented in a manner more easily understood. Therefore, the theory presentation below will be adapted from both Heard's 1976 and 1978 efforts.

Heard begins by assuming a collection of particles in a common force field with the following equations of motion:

$$g = \underline{X}(g,p,t), \quad p = \underline{Y}(g,p,t) \tag{7}$$

where the vectors g and p represent the coordinates and momenta of the particles, respectively. The next assumption is that f(g,p,t) is the phase-space density function such that a volume contains dN particles with the following relation:

$$dN = f(g,p,t) \, dg \, dp \tag{8}$$

Heard then refers to Chandresekhar's solution that says the distribution function f follows the relationship

$$df/dt = \sum_{i=1}^{n} (d/dq_i \cdot (fX_i) + d/dp_i \cdot (fY_i)) = 0 \tag{9}$$

if no source of particles is present. This equation is then rewritten in the following form:

$$Df/Dt = -f\Delta \tag{10}$$

where D/Dt is the "Stokes derivative" (D/Dt = d/dt + $\underline{X} \cdot \nabla g$ + $\underline{Y} \cdot \nabla p$) and $\Delta = \sum_{i=1}^{n} (dX_i/dq_i + dY_i/dp_i)$.

The next step is to establish the position (g,p) of a particle at time t = 0. Heard lets $\underline{Q}_0(g,p,t)$, $\underline{P}_0(g,p,t)$ be

this position. Similarly, $\underline{Q}(g,\underline{p},t)$, $\underline{P}(g,\underline{p},t)$ is the particle's position at time t.

Heard now shows the solution of Eq (10), given the initial condition

$$f(g,\underline{p},0) = F(g,\underline{p}) \tag{11}$$

to be

$$f(g,\underline{p},t) = F(\underline{Q}_0(g,\underline{p},t),\underline{P}_0(g,\underline{p},t)) \exp\{-\Gamma(g,\underline{p},t)\} \tag{12}$$

where

$$\Gamma(g,\underline{p},t) = \int_0^t \Delta(g(t),\underline{p}(t),t) \, dt \tag{13}$$

The integral in Eq (13) is to be evaluated on the trajectory which passes through $(g,\underline{p})$ at time t

The spatial density $\rho(g,t)$ is found by integrating f over all momenta as follows:

$$\rho(g,t) = \int f(g,\underline{p},t) \, d\underline{p} \tag{14}$$

From this equation, the shape and density of the cloud can be found at any time.

Next, Heard finds the solution given dispensing from the initial point g*. The initial condition

$$F(g,\underline{p}) = \delta(g - g*) \, G(\underline{p}), \tag{15}$$

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applies. Here, $\delta(q - q*)$ is the Dirac delta-function and
$G(p)$ is any function that describes the initial distribution
of the particle momenta. Heard next applies theorems
relating to the δ-functions to arrive at the following
equation to substitute for Eq (14):

$$\rho(q,t) = 1/|J| \times G(\underline{P}_0(q,\underline{p}*,t)) \exp\{-\Gamma(q,\underline{p}*,t)\} \tag{16}$$

where $\underline{p}*$ is the solution of

$$\underline{Q}_0(q,\underline{p}*,t) = q* \tag{17}$$

and $|J|$ is the Jacobian determinant

$$J = \frac{d(Q_{0,1},\ldots,Q_{0,n})}{d(p_1,\ldots,p_n)} \tag{18}$$

Heard next provides matrix equations to describe the
particle motions. When the previous results are combined
with the matrix equations, a general solution to the problem
can be found.

First, Heard lets q and p represent small deviations
from the reference solution of Eq (7). The variational
equations of motion for q and p become

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = A \begin{pmatrix} q \\ p \end{pmatrix} \tag{19}$$

where A is the matrix

$$A = \begin{pmatrix} A11 & A12 \\ A21 & A22 \end{pmatrix}$$

Here A11 = $d\underline{x}/dg$, A12 = $d\underline{x}/dp$, A21 = $d\underline{y}/dg$, and A22 = $d\underline{y}/dp$. All are evaluated on the reference solution. When Heard solves the variational equations, he uses the matrizant

$$\Phi = \begin{pmatrix} U & V \\ W & Y \end{pmatrix} \tag{20}$$

where

U = dg/dg_0, V = dg/dp_0, W = dp/dg_0, ε Y = dp/dp_0.

The solution then becomes

$$\begin{pmatrix} g \\ \underline{p} \end{pmatrix} = \Phi \begin{pmatrix} g_0 \\ \underline{p}_0 \end{pmatrix} \tag{21}$$

where g_0 and $\underline{p}_0$ are the initial values.

Heard next adopts the following matrix notation:

$$M_-(t) = M(-t)$$

and notes that for a matrizant the following property holds:

$$\Phi^{-1} = \Phi_- \tag{22}$$

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From Eqs (20) and (21) the following holds true:

$$\begin{pmatrix} q_0 \\ p_0 \end{pmatrix} = \Phi_- \begin{pmatrix} q \\ p \end{pmatrix} \tag{23}$$

q_0, p_0, and J end up in the following forms:

$$\dot{q}_0(q,p,t) = U_- q + V_- p \tag{24}$$

$$\dot{p}_0(q,p,t) = W_- q + Y_- p \tag{25}$$

$$J = \det(V_-) \tag{26}$$

Eq (17) now takes on the following form:

$$U_- q + V_- p* = g* \tag{27}$$

with a solution of

$$p* = V_-^{-1}(g* - U_- q) \tag{28}$$

Now, Heard lets $L = \det(\Phi)$ and differentiates L, with
the solution being

$$dL/dt = L \sum_{i=1}^{n} (d\dot{q}_i/dq_i + d\dot{p}_i/dp_i) \tag{29}$$

From this, Heard ultimately obtains the final expression for
the exponent term in Eq (16) using

$$dL/dt = L\Delta \tag{30}$$

which becomes

$$d/dt(\ln L) = \Delta \tag{31}$$

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to give

$$\exp\{-\int_0^t \Delta dt\} = L(0)/L(t) \tag{32}$$

By using Eqs (25), (26), (28), and (32), substituting into Eq (16) and using the fact that $\det(\Phi) = 1$ according to Liouville's Theorem, Heard's solution becomes

$$\rho(\underline{g},t) = \{1/|\det(V_-)|\} \{G[(W_- - Y_-V_-^{-1}U_-)g + Y_-V_-^{-1}g*]\} \tag{33}$$

The last step Heard uses is to apply the previous results to the specific case of a group of particles under the influence of the force of gravity and orbiting about the earth. Heard's solution uses the cylindrical coordinates $(\bar{w},\theta,z)$ and assumes a radially symmetric gravitational potential (v). The coordinates are set up with $z = 0$ in the equatorial plane, so v becomes a function of only $\bar{w}$ and z. An orbit of radius R can be described in this coordinate system as follows:

$$\bar{w} = R, \quad \theta = \theta_0 + \Omega t, \text{ and } z = 0 \tag{34}$$

where $R\Omega^2 = dv/d\bar{w}(R,0)$ and Ω is the angular velocity of the parent satellite. Small changes in the orbit are described by ξ, η, and ζ in the following manner:

$$\bar{w} = R + \xi, \quad \theta = \theta_0 + \Omega t + \eta/R, \text{ and } z = \zeta \tag{35}$$

Next, Heard uses the Hamiltonian

$$H = (1/2)(p1^2 + p2^2 + p3^2) - 2\Omega\xi p2 + (1/2)(n^2\xi^2 + K^2\zeta^2) \quad (36)$$

with momenta

$$p1 = \dot{\xi} , \quad p2 = \dot{\eta} + 2\Omega\xi , \quad \text{and} \quad p3 = \dot{\zeta} \quad (37)$$

and where

$$n^2 = 3\Omega^2 - (d^2v/d\bar{w}^2)(R,0) \quad \text{and} \quad K^2 = (d^2v/dz^2)(R,0) \quad (38)$$

In Eqs (36) and (38), n is the epicyclic frequency. Both n^2 and K^2 are assumed to be positive.

Using the following notation:

$$\mathcal{T} = nt, \quad \mathcal{T}' = Kt, \quad \sigma = 2\Omega/n, \quad s = \sin\mathcal{T},$$
$$c = \cos\mathcal{T}, \quad s' = \sin\mathcal{T}', \quad \text{and} \quad c' = \cos\mathcal{T}'$$

Heard then arrives at the fundamental matrices for U, V, W, and Y for use in Eq (33)

$$U = \begin{pmatrix} c & 0 & 0 \\ -\sigma s & 1 & 0 \\ 0 & 0 & c' \end{pmatrix}$$

$$V = \begin{pmatrix} s & (1 - c) & 0 \\ \sigma(c - 1) & (1 - \sigma^2)\mathcal{T} + \sigma^2 s & 0 \\ 0 & 0 & ns'/K \end{pmatrix}$$

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$$W = \begin{pmatrix} -ns & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -Ks' \end{pmatrix}$$

$$Y = \begin{pmatrix} c & \sigma s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c' \end{pmatrix}$$

In order to simplify the solution, he also establishes the following convenience:

$$D = [(1 - \sigma^2)s + 2\sigma^2(1-c)] \qquad (39)$$

Finally, Heard obtains an equation which describes the density function for the collection of particles using any initial momentum distribution function $G(p1,p2,p3)$. The equation is as follows:

$$\rho(\xi,\eta,\zeta,t) = \{K/|s'[(1 - \sigma^2)s\tau + 2\sigma^2(1 - c)]|\}$$
$$\times\ G\Big((n/D)\{[(1 - \sigma^2)\xi +\sigma^2 s]\tau - \sigma(1 - c)\eta\},\ (40)$$
$$(n/D)[\sigma(1 - c)\xi + s\eta],\ [(K/s')\zeta]\Big)$$

where $\sigma = 2$, and $n = K = \Omega = 1$ for Keplerian orbits.

From this equation, the distribution of particles in the chaff cloud at any future point in time can be determined at any point (ξ,η,ζ) with reference to the parent satellite. Since the center of mass of the cloud follows

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that of the original orbit and the orbit of the parent

satellite is known, the motion of the chaff cloud can be

easily determined.

Attenuation Through the Chaff Cloud

The final section of this chapter takes a look at the

basic theory describing the attenuation of an

electromagnetic signal as it penetrates an absorbing medium

such as a chaff cloud. Although the attenuation analysis

could be via either wave or particle theory, only particle

theory is examined. The primary reason for this one-sided

approach is that Brown used particle theory in his work with

satisfactory results; an analysis that applied wave theory

would be needless duplication of effort.

Chaff Cloud Attenuation Theory. Brown states that a

cloud of chaff particles affects an electromagnetic signal

by first receiving and absorbing an incident signal and then

re-radiating the signal at some later time. This

re-radiation can occur in effectively any direction with the

overall effect of uniformly scattering over 4π steradians

that portion of the signal which was initially absorbed. By

placing enough dipoles in random orientations into the path

of the incoming signal, the signal can be attenuated

sufficiently to effectively impede transmission through the chaff cloud (Brown, 1987:4-1 to 4-2).

The chance of a dipole interacting with the incoming signal is directly determined by the "effective cross-section" of the dipole. The effective cross-section is a measure of the probability of interaction between a dipole and the signal rather than the true cross-sectional area measurement of the dipole as the name implies. Nonetheless, the effective cross-section is given in units of area (Brown, 1987:4-2).

The effective cross-section is dependent on the frequency of the signal and the length of an individual dipole. A maximum value for effective cross-section occurs when the length cf the dipole equals one-half the wavelength of the signal. The value of effective cross-section (σ) for a group of randomly oriented dipoles can be determined from Eq (41) below:

$$\sigma = 0.16 \times \lambda^2 \qquad (41)$$

where λ is the wavelength of the signal (Brown, 1987:4-2 to 4-3; Peebles, 1984:128-129).

According to Brown, the effect of a uniform chaff cloud on the entire signal is determined from Eq (42)

$$P_{out}/P_{in} = \exp\{-(\sigma z N/V)\} \qquad (42)$$

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Here, P_{out}/P_{in} is the decimal fraction of signal power

transmitted through the cloud, σ is the effective cross-

section from Eq (41), z is the distance the signal travels

through the chaff cloud, N is the number of chaff particles

in a particular volume of space, and V is the volume in

which the cloud is contained (Brown, 1987:4-13 to 4-14).

According to Evans, it is essential that the chaff cloud be

uniform in order for Eq (42) to provide accurate results

(Evans, 1988). Therefore, when the chaff cloud is modeled,

it has to be broken down into subvolumes that are small

enough for uniform density to be present.

Once the desired degree of signal attenuation is

established, the appropriate value for 'zN/V' to be used in

Eq (42) is easily calculated. Since the value for 'z'

through any part of the cloud should be known, the value of

'N/V' (particle density) required for signal attenuation is

also determinable.

The beamwidth of the signal at the altitude of the

chaff cloud is also important since any signal which goes

around the edges of the cloud is not attenuated. Therefore,

the effective attenuation of the signal does not occur until

the chaff cloud reaches a size in area at least as large as

that of the incident signal (Brown, 1987:4-3 to 4-6). This

last statement also assumes enough chaff particles are at

the edges of the chaff cloud so that effective attenuation

occurs. In practice, the cloud has to be much greater in size than this minimum so that the number density in the attenuation region is high. This is because of the cloud having a smaller density at its edges and a higher density near the centroid.

Finally, the above values for 'z', 'N/V', and signal attenuation are calculated for different points in time after initial deployment of the cloud. These values are used to establish the required dispensing velocity of the particles in order to affect the desired attenuation at any given point in time after deployment. From these values, the period of time before effective attenuation occurs and the period of time over which attenuation takes place are found (Brown, 1987:4-13 to 4-17).

III. Methodology

This chapter outlines the approach followed to complete the chaff cloud study and is designed as only a very basic precursor of what follows in Chapters IV, V, and VI. The chapter is divided into the following sections: application of Heard's results, graphical presentation of new results, and determining the overall effectiveness of the chaff cloud as a communications jamming device.

Applying Heard's Results

The methodology used in the application of Heard's results to the chaff cloud problem can be broken down into five sections. The five sections are 1) a dimensional analysis of Eq (40) to ensure complete understanding, 2) determining the appropriate velocity distribution to be applied to the chaff cloud expansion, 3) determining particle density throughout the chaff cloud using Eq (40), 4) a spreadsheet setup for ease of calculation of results, and 5) attenuation calculations of a signal through the chaff cloud using Brown's thesis as a basis.

Dimensional Analysis. The first thing done with Heard's analysis was to break down Eq (40) into its component parts. These parts were then subjected to a

dimensional analysis to determine the scaling and
normalization factors used by Heard in his derivation. The
components of the equation were next reassembled and solved
using the example velocity distribution function provided by
Heard. The values for (ξ, η, ζ, t) obtained in this manner
could then be plotted on an η versus ζ graph and compared to
η versus ζ plots in Heard's paper to ensure the equation was
being solved properly.

Determining the Velocity Distribution Function. Once
Eq (40) was fully understood, a solution using a velocity
distribution function appropriate to the chaff cloud problem
was needed. Since the other portions of this analysis
involve a statistical approach rather than strictly
numerical solutions, a statistical approach was also applied
to the determination of the velocity distribution function.
With the distribution of particles being approximately
Gaussian, the distribution function arrived at through this
approach was Maxwellian.

Determining Particle Density. Eq (40) allows a
reference value for particle density to be calculated at any
selected point within the chaff cloud. By summing each of
these reference values from points throughout the cloud, an
aggregate total against which each of the individual values
can be compared as a ratio is obtainable. These ratios can

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then be multiplied by the number of chaff particles in the entire cloud to convert the ratios into the actual number of chaff particles at each point within the cloud.

The particle density of a particular volume within the cloud can be approximated by the density at the center of the volume if the volume is small enough that major variations across the volume are eliminated. This was accomplished for the chaff cloud by dividing the cloud into small cubic volume elements and then following the ratio procedure from above to determine the number of particles within each volume element.

Spreadsheet Setup. In order to allow the many iterations required for this analysis to be accomplished more easily, a spreadsheet divided into four sections was used to do the calculations. The first section included constants and variables to be used in the later sections with the variables including time and dispensing velocity. The second section consisted of Eq (40) broken up into several parts for ease of calculation. The next section summed up the relative densities mentioned above, determined particles ratios, and calculated the number of particles within volume elements located throughout the cloud. The final section of the spreadsheet contained the calculations for attenuation of the signal through the chaff cloud.

Attenuation of the Incident Signal. Using the number of particles in the volumes along each signal path from above and the values provided by Brown in his thesis for the required size and altitude of the chaff cloud, the values for attenuation of the incident signal at various points across the cloud's cross-section (as seen from the earth) were obtained. These data were compiled for each of several different dispensing velocities and run for the time interval beginning with initial cloud dispensing and ending 12 hours later. The results were then presented in the graphs outlined in the next section to describe the signal attenuation affected by the chaff cloud.

Graphical Presentation

Once results for various dispensing velocities and times after initial dispensing were obtained, it was necessary to present the results in a useful format. It was decided to provide a pictorial representation of the chaff cloud attenuation through plots of dispensing velocity versus signal attenuation. Using different symbols to represent different amounts of time elapsed after cloud dispensing took place, the signal attenuation through the chaff cloud as a function of both time and dispensing velocity was determined.

Several important parameters were obtained from these graphs. First, the amount of time elapsed before effective attenuation occurs could be determined for any given dispensing velocity. Second, the length of the period of effective attenuation for a given dispensing velocity was available. This was possible since the attenuation plots were run past the time at which the attenuation levels drop below effective levels. Finally, a comparison between the various dispensing velocities was made based on specific requirements for signal attenuation. These comparisons were based on the amount of time before effective attenuation, the period of time over which effective attenuation occurs, and the amount of time which elapses before attenuation is no longer effective.

Effectiveness of Attenuation

The final step in the analysis was to determine whether the attenuation levels obtained by the chaff cloud are effective when the following questions were asked:

1) Are the attenuation levels attained by the cloud both high enough and maintained long enough for a chaff cloud to be of value?

2) How does the fact that the chaff cloud and the satellite to be jammed are moving at different velocities affect signal attenuation?

3) Given the initial size, shape, makeup, and weight of the ball of chaff, are the assumptions surrounding the dispensing of the chaff cloud reasonable?

4) What are the effects of the chaff cloud on satellites other than the one the chaff cloud is intended to jam?

IV. Application of Heard's Results

This chapter applies the methodology and theory previously outlined to the chaff cloud problem. This allows an overall attenuation picture to be obtained. The chapter is divided into the following sections: 1) dimensional analysis of Heard's final equation, 2) determination of a velocity distribution function, 3) determination of particle densities throughout the chaff cloud, 4) the spreadsheet setup used for analyzing Heard's equation, and 5) a numerical analysis of signal attenuation using Brown's thesis as a basis.

Dimensional Analysis

In order to simplify his calculations, Heard normalized to unity some of the parameters used in his analysis through the use of convenient dimensions. The two major parameters set to unity were the radius of the parent satellite's orbit and the satellite's mean motion.

Satellite Radius. The radius of a satellite's orbit is nined by adding the satellite's altitude above the earth to the earth's radius. Therefore, the parent satellite for the chaff cloud, which is 100 nautical miles (NM) below a geosynchronous satellite, has an orbital radius

of approximately 42,073 kilometers (km) (Brown, 1987:4-4;
Illustrated_Encyclopedia_of_Space_Technology, 1982:86).
Heard normalized the orbital radius of the parent satellite
of the chaff cloud to unity using what will be referred to
in the remainder of this paper as Radius Units (RU).
Therefore, in the case of the parent satellite,

$$1 \text{ RU} = 42073 \text{ km} \qquad (43)$$

Mean_Motion. A satellite's mean motion describes the
time it takes for the satellite to travel through one radian
of its orbit. Next, a Satellite Time Unit (STU) is defined
so that a Radius Unit per Satellite Time Unit (RU/STU) is
the velocity of a satellite. When an RU/STU is set to
unity, then the value of 'GM' in Eq (44) below also becomes
unity with dimensions of RU^3/STU^2:

$$Vcs = (GM/a)^{1/2} \qquad (44)$$

where Vcs is the satellite's velocity, and a is the
semi-major axis as defined in Chapter II (Bate and others,
1971:34). For the parent satellite, 1 RU/STU has a value of
3078 meters per second (m/s).

Using Eq (45) below, 'n' also acquires a value of unity
with units of radians per STU (rad/STU):

$$n = (GM/a^3)^{1/2} \qquad (45)$$

(Bate and others, 1971:185).

The conversion between STUs and conventional time units is accomplished by dividing a satellite's orbital period by 2π radians (rad). For the parent satellite, the calculation is as follows:

$$1 \text{ STU} = 85880 \text{ sec}/2\pi \text{ rad}$$
$$= 13670 \text{ sec} \tag{46}$$

Velocity Distribution Function

The work of Knott, Lewinski, and Hunt suggests that a spherical chaff cloud should have approximately a Gaussian particle distribution (Knott, 1981:3,112). If a Gaussian particle distribution is assumed, the appropriate velocity distribution function should be Maxwellian (Evans, 1988). A Maxwellian distribution function has the following form (Evans, 1987:10)

$$n_v(z,v) = n(z)(2/\pi)^{1/2}(v^2/V_0^3)\exp(-v^2/2V_0^2) \tag{47}$$

where $v^2 = (v_\xi^2 + v_\eta^2 + v_\zeta^2)$, and V_0 represents the "thermal speed" of the particles. The thermal speed of the chaff particles is defined as being equal to the dispensing velocity and is assumed to be known.

A new term, the most probable particle velocity (V_{th}), is now defined by Eq (48)

$$V_{th} = \sqrt{2}\, V_0 \tag{48}$$

Values of V_{th} for several values of V_0 are shown in Table 1.

Table I. Most Probable Particle Speeds

for Various Dispensing Velocities

V_0 (m/s)	V_0 (RU/STU)	V_{th} (RU/STU)	V_{th}^2 (RU²/STU²)
0.0050	1.624×10^{-6}	2.297×10^{-6}	5.275×10^{-12}
0.0100	3.249×10^{-6}	4.595×10^{-6}	2.107×10^{-11}
0.0120	3.899×10^{-6}	5.514×10^{-6}	3.040×10^{-11}
0.0130	4.224×10^{-6}	5.973×10^{-6}	3.568×10^{-11}
0.0140	4.548×10^{-6}	6.432×10^{-6}	4.138×10^{-11}
0.0150	4.873×10^{-6}	6.891×10^{-6}	4.749×10^{-11}
0.0200	6.498×10^{-6}	9.190×10^{-6}	8.446×10^{-11}
0.0250	8.122×10^{-6}	1.149×10^{-5}	1.320×10^{-10}
0.0300	9.746×10^{-6}	1.378×10^{-5}	1.899×10^{-10}
0.0350	1.137×10^{-5}	1.608×10^{-5}	2.586×10^{-10}
0.0400	1.300×10^{-5}	1.838×10^{-5}	3.378×10^{-10}
0.0450	1.462×10^{-5}	2.068×10^{-5}	4.277×10^{-10}
0.0500	1.624×10^{-5}	2.297×10^{-5}	5.275×10^{-10}

Now, the momenta are generalized by dividing through by particle mass, and are substituted with Eq (48) into Eq (47). This results in the distribution function shown in Eq (49) below:

$$G(p) \sim \exp(-(p1^2 + p2^2 + p3^2)/V_{th}^2) \qquad (49)$$

Determining Particle Density

The distribution function from the previous section is now used in conjunction with Eq (40) to determine a reference value for the number of chaff particles at any given (ξ, η, ζ) coordinates and at any given time after initial cloud dispensing. Once the reference number is known at locations throughout the cloud, the actual number of particles at these same locations is also determinable.

First, the reference values at each location are summed to obtain the reference total. Then, the reference number at each location is divided by the reference total to determine the decimal fraction of the total number of chaff particles at each location. Finally, by multiplying each of these fractions by the total number of chaff particles, the actual number of particles at each location is found.

Since it would be impossible to calculate reference numbers for every location in space, the cloud is broken up into a three-dimensional grid of identically sized cubes

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with the centroid (origin) of the innermost cube at the
(ξ, η, ζ) coordinate $(0,0,0)$. The grid has a size of seven
units on a side with the units for the overall grid being of
variable size for the purpose of scaling the model to fit
the chaff cloud as it expands over time. The 7x7x7 grid
thus allows for a chaff cloud of size up to plus and minus
three and one-half units of the initial dispensing point in
each of the three orthogonal directions. The final size of
the grid is set at 7x7x7 because of memory limitations (640
kilobytes of Random Access Memory) in the computer used to
run the program. The limitation on the number of grid cubes
in which to contain the expanding cloud is counteracted by
the scaling capability of the program, however.

With the grid in place, the reference values need to be
calculated only at the centroids of each cube. Each of the
reference values can now be assumed to represent the
reference number of particles within each of these cubic
subvolumes of ·ace instead of only at particular
coordinates. Therefore, when the actual number of particles
at each centroid is determined, the particle densities at
the various locations are easily determined by dividing the
number of particles by the volume contained in the
subvolumes.

One obvious difficulty with the above approach is that
to obtain accurate results, the particle density in each

subvolume must be uniform. In the actual chaff cloud, this clearly might not be the case. However, the use of variable cube sizes for the scaling process also allows for improved internal density resolution of the chaff cloud.

If the uniformity of the particle density in any one cube is suspect, the centroid of the entire grid can be moved from (0,0,0) to the coordinates of the centroid of the appropriate cube. The sizes of the subvolumes within the suspect cube can then be reduced. In this manner, any of the original subvolumes can be "magnified" and subsequently examined for internal particle density uniformity. Figure 7 shows this process pictorially.

Another disadvantage of the grid approach is that while the cloud expands radially to form a sphere and then an ellipsoid, the model employs a cubic grid to examine the cloud. Again, by moving the origin of the entire grid to the various parts of the chaff cloud ; , using the scaling technique described above, the actual shape of the cloud can be closely approximated at any point in time.

An advantage of the grid approach is that once the number of particles in each grid cube throughout the entire cloud is known, signal attenuation through any particular portion of the cloud can be determined independently of the attenuation through any other portion of the cloud. An example is examining the attenuation of a signal which
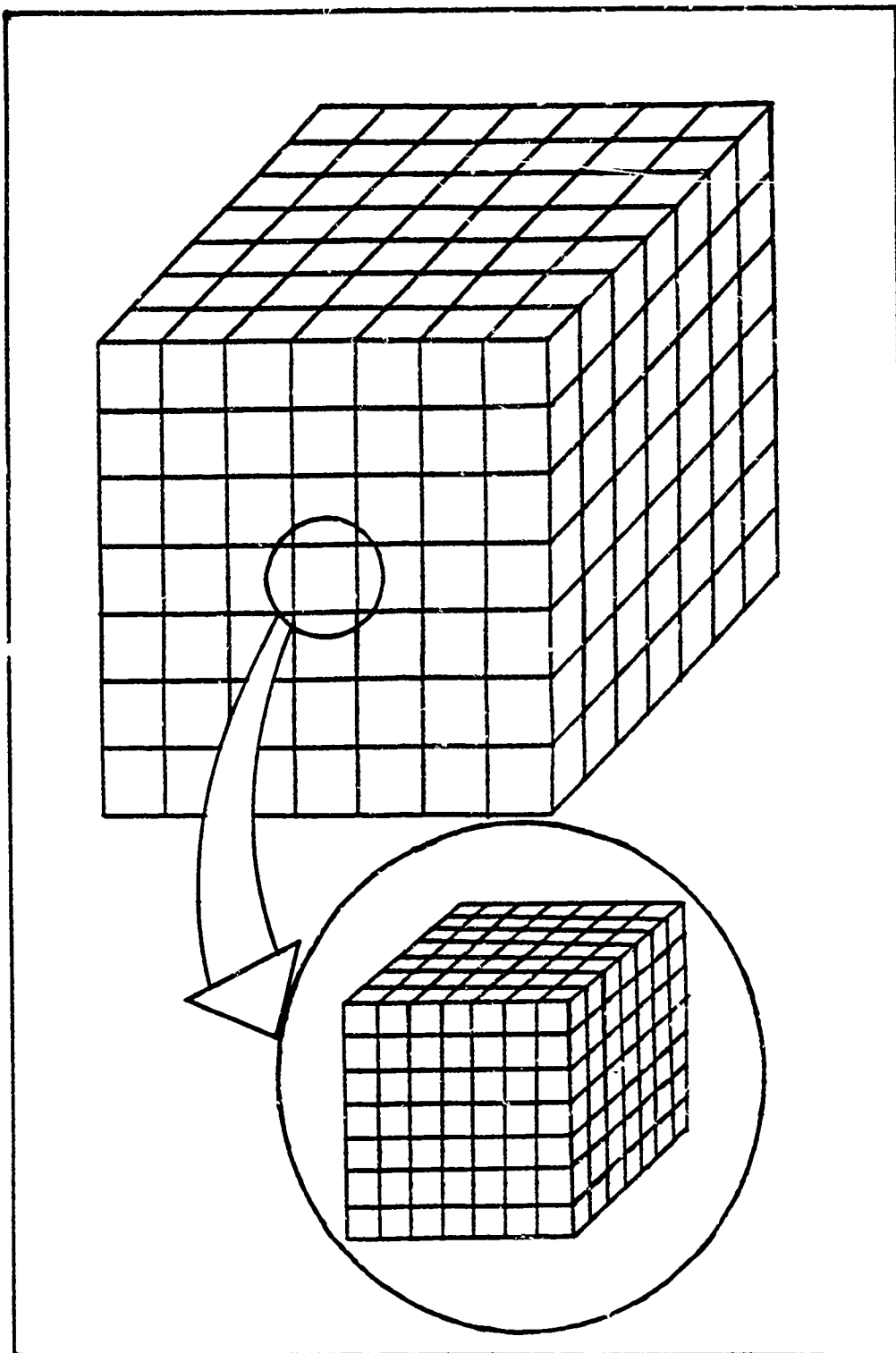
Figure 7. "Magnification of a Subvolume
of the Chaff Cloud

penetrates only through that portion of the cloud within plus or minus one kilometer in the η and ξ directions of the center of the cloud. This advantage will become more evident later in the section concerning Brown's attenuation requirements for a chaff cloud.

Spreadsheet Setup

The attenuation of the electromagnetic signal is calculated using two spreadsheet programs. The first spreadsheet calculates the number density in each cube of the grid. The second calculates the signal attenuation through the cloud starting at subvolumes across the face of the cloud and continues by looking at subvolumes at successive depths (values of ξ) through to the other side. Each run of the spreadsheet programs constitutes one point in time after cloud dispensing began. The spreadsheet programs were devised using Quattro by Borland and are included as Appendices A and B, respectively.

The number density spreadsheet is broken up into three sections. The first provides basic equations and constants, the second calculates reference number densities for the subvolumes in the grid, and the third section calculates actual number densities in each subvolume. A more detailed explanation of the spreadsheet is included as part of Appendix A.

The signal attenuation spreadsheet is divided into two sections. The first is composed of all the values from section three of the number density spreadsheet. In practice, these values are first copied as a block into a separate file and then transported into the signal attenuation spreadsheet. The second section of this spreadsheet uses the values from section one to calculate the attenuation of the incident signal (P_{in}). The attenuation calculations determine the decimal fraction of the signal that penetrates through each of 49 columns of the chaff cloud (P_{out}). The setup and numbering scheme of the 49 columns is shown in Figure 8, with the horizontal and vertical directions corresponding to the η and ζ directions, respectively. For each column, Eq (42) is applied to the number of particles contained within each subvolume; the values at each layer are multiplied together to determine the overall value for 'P_{out}'. Care is taken to ensure the appropriate values are used for 'z' and 'V' to compensate for the various magnifications being used in the spreadsheet.

Attenuation Analysis Using Brown's Thesis as a Basis

Brown's thesis shows that the minimum required area of a chaff cloud attempting to jam communications between a DSCS satellite and a circular area the size of Washington

(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)	(7,1)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)	(7,2)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)	(7,3)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)	(7,4)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)	(7,5)
(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)	(7,6)
(1,7)	(2,7)	(3,7)	(4,7)	(5,7)	(6,7)	(7,7)

Figure 8.   Numbering Scheme for the Subvolumes
Along the Central Column of the Chaff Cloud

D.C. is approximately 97,500 square meters. This assumes that the chaff cloud is located 100 NM (185.2 km) below the DSCS satellite, an 8 Gigahertz (GHz) uplink signal is being jammed, and that the cloud contains 100 billion particles (Brown, 1987:4-3 to 4-5). This equates to a chaff cloud with a required radius of approximately 175 meters (m) and a diameter of 350 m. Brown also states the cloud must reach an attenuation of at least -10 decibels (dB) across the entire area of coverage within one hour of dispensing and must maintain this -10 dB attenuation for at least twelve hours (Brown, 1987:4-14 to 4-17). Stanley's book Electronic Communications Systems provides a good explanation of attenuation calculations using decibels (Stanley, 1932:7-9).

   With the area of coverage on the ground for the chaff cloud being circular, the only portion of the chaff cloud important to signal attenuation is the column which runs through the centroid of the cloud. Therefore, only attenuation through this part of the cloud needed to be calculated.

   When a circle with a radius of 350 m is inscribed in a cross-section of the spreadsheet model grid with dimensions of 350 m, the squares at the corners of the grid lie entirely outside of the circle. As a result, only 45 of the 49 columns contained within the model needed to be examined for signal attenuation. This is shown in Figure 9.
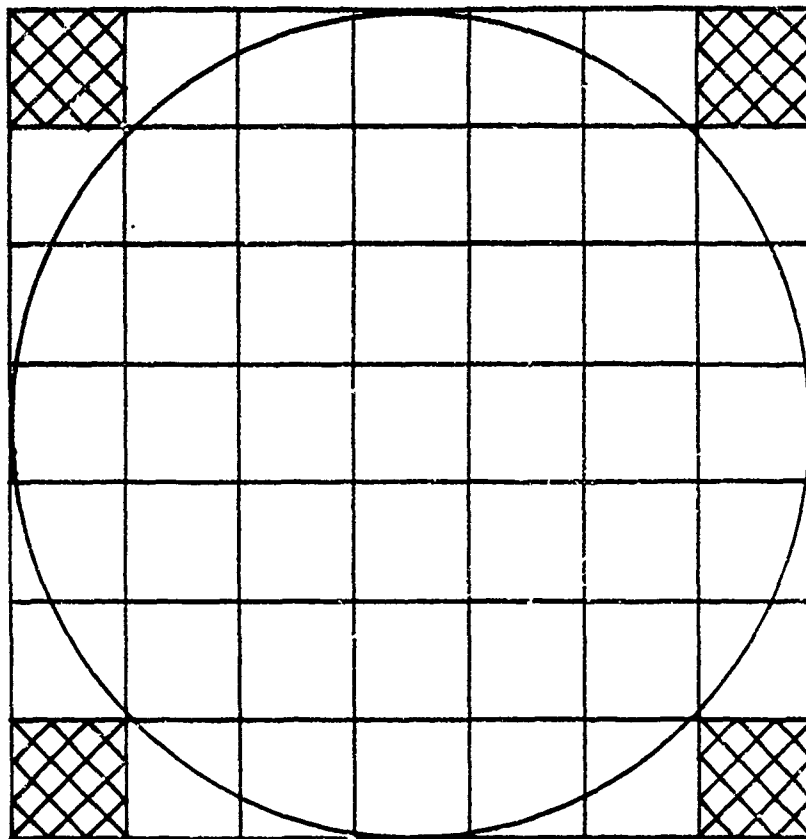
Figure 9.   Effect of Inscribing a Circle
Within the Chaff Cloud Model

To determine if Brown's first attenuation requirement is met by a chaff cloud in orbit, the entire cloud is first examined at a time of one hour after dispensing, with the size of the internal grid cubes increased as required to ensure all chaff particles remain within the 7x7x7 model. The number of chaff particles within all of the subvolumes are then recorded. Then, the cloud model is scaled back down a step at a time until grid cubes 50 m on a side are obtained. It is important to assign the proper number of chaff particles to each of the subvolumes and the proper coordinates to the origin of the grid each time the model is scaled up or down. The attenuation of the signal is then determined using the attenuation spreadsheet. By varying the dispensing velocities and repeating the above process, the minimum dispensing velocity which meets the criterion for attenuation within one hour is established.

In order to determine whether a -10 dB attenuation is maintained for a period of twelve hours and, if not, for how long it is maintained, a similar approach is followed. The spreadsheets are run at one hour increments for the following dispensing velocities (all in m/s): 0.01, 0.02, 0.03, 0.04, and 0.05. Again, the entire cloud is kept within the model's grid through appropriate scaling of the spreadsheets for each pair of velocity and time values.

The cloud is then scaled back down as required until the 350 meter diameter cloud is reached. The number of particles within each subvolume are recorded at each step. These values are then used in subsequent steps as the total number of particles contained in the 7x7x7 subscaled grids.

The downscaling process involves first determining the number of particles in each of the seven $(\eta, \zeta) = (4,4)$ subvolumes in the total cloud model. Each of these subvolumes is then run through the density spreadsheet individually. This is accomplished by rescaling the spreadsheet so the outer dimensions of the subvolume exactly fit the 7x7x7 grid, using the coordinates of the subvolume centroids as the origin for the subscaled spreadsheet grid, and ensuring the number of particles within the subvolume is entered into the "constants" section of the spreadsheet. By using this process repetitively, the number of particles in each sub-subvolume is determined until the number of particles within each layer of the central column is determined. Finally, by manually applying Eq (42) through each individual layer of the central column, and multiplying the final results together, the overall attenuation affected by the cloud is obtained.

In order to clarify the above discussio: . ppendix C describes the procedures followed for one attenuation calculation. The values used in the example for dispensing

velocity and time after dispensing are 0.05 m/s and six
hours, respectively.

V. Presentation of Results and Discussion

This chapter presents the results for each of the areas of this research effort and an interpretation of these results. The chapter is divided into the following three sections with interpretations included as a part of each section: physical structure of the chaff cloud over time, signal attenuation one hour after dispensing, and signal attenuation for the first twelve hours after dispensing.

Physical Structure of the Chaff Cloud

As would be expected from the discussion in Chapter II, the chaff cloud's physical structure closely resembles the cloud described for Case II since it is dispensed instantaneously and has velocity components both along the path of motion and radially outward. The only real change should be that some of the chaff particles also have velocity components out of the orbital plane.

The cloud starts out as a sphere and exhibits a gradual spreading out in all directions. The spreading becomes faster for the higher dispensing velocities, but the general shape of the cloud remains similar in each case. Figures 10, 11, 12, and 13 show the cloud evolution over time in the η and ζ directions for a 0.01 m/s dispensing velocity.
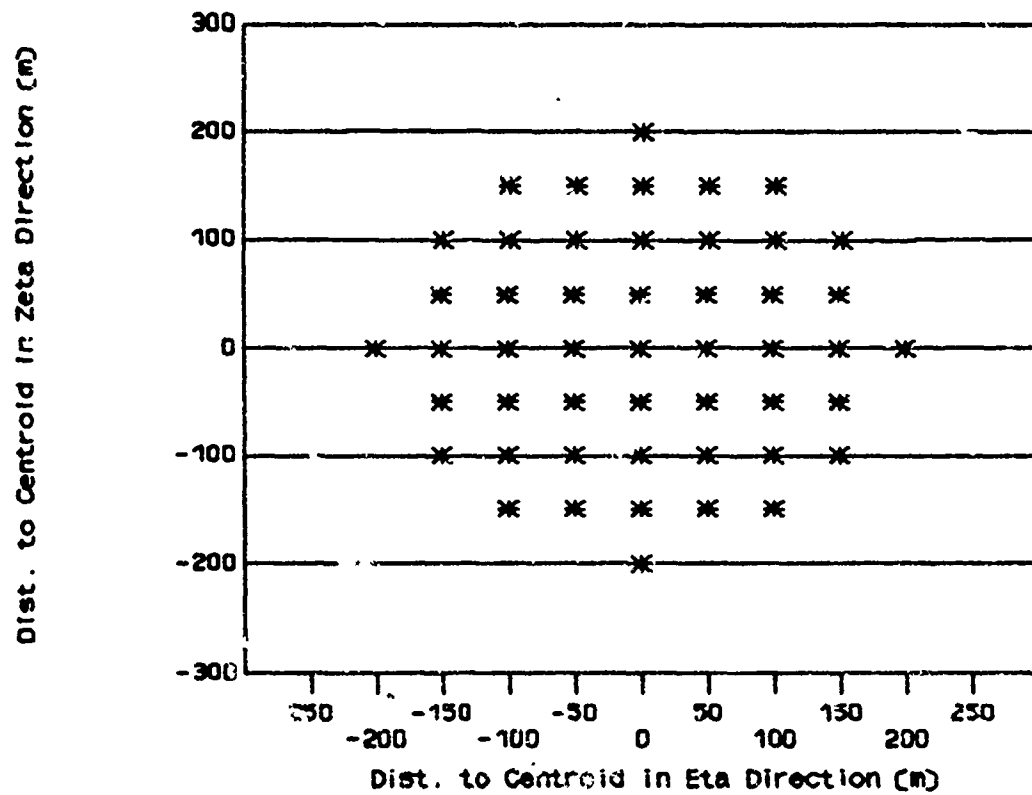
Figure 10.   ._ff Cloud Evolution for a 0.01 m/s Dispensing
               elocity: 3600 Seconds After Dispensing
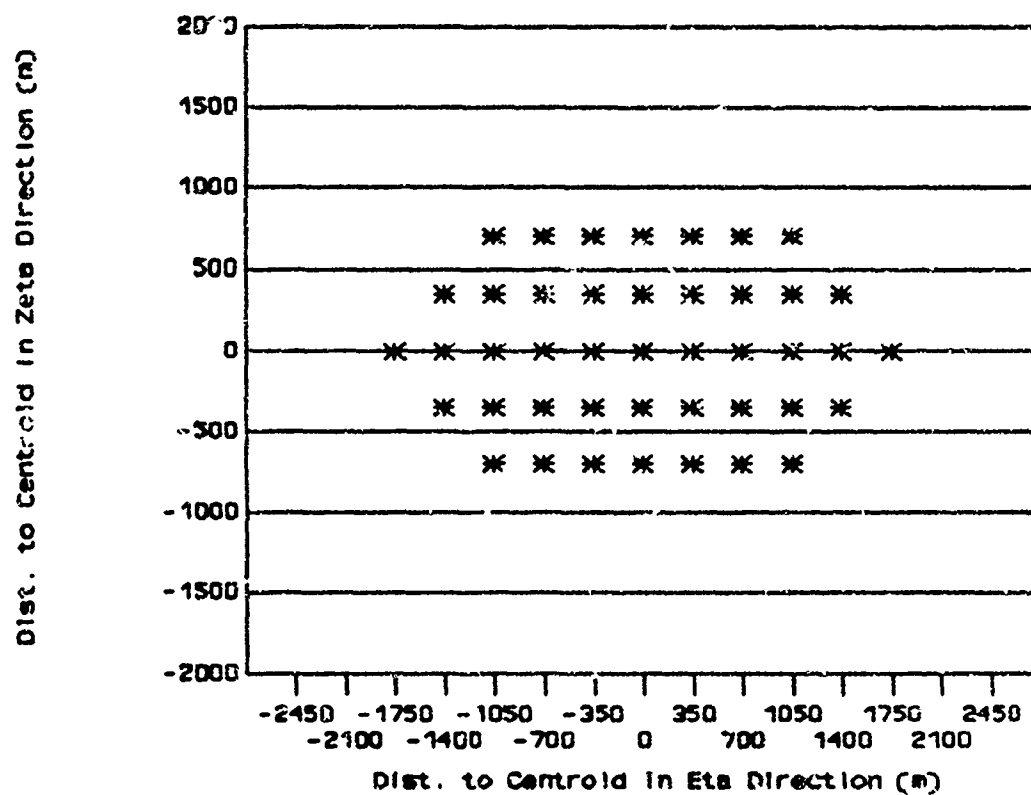
Figure 11.   Chaff Cloud Evolution for a 0.01 m/s Dispensing
             Velocity: 25200 Seconds After Dispensing
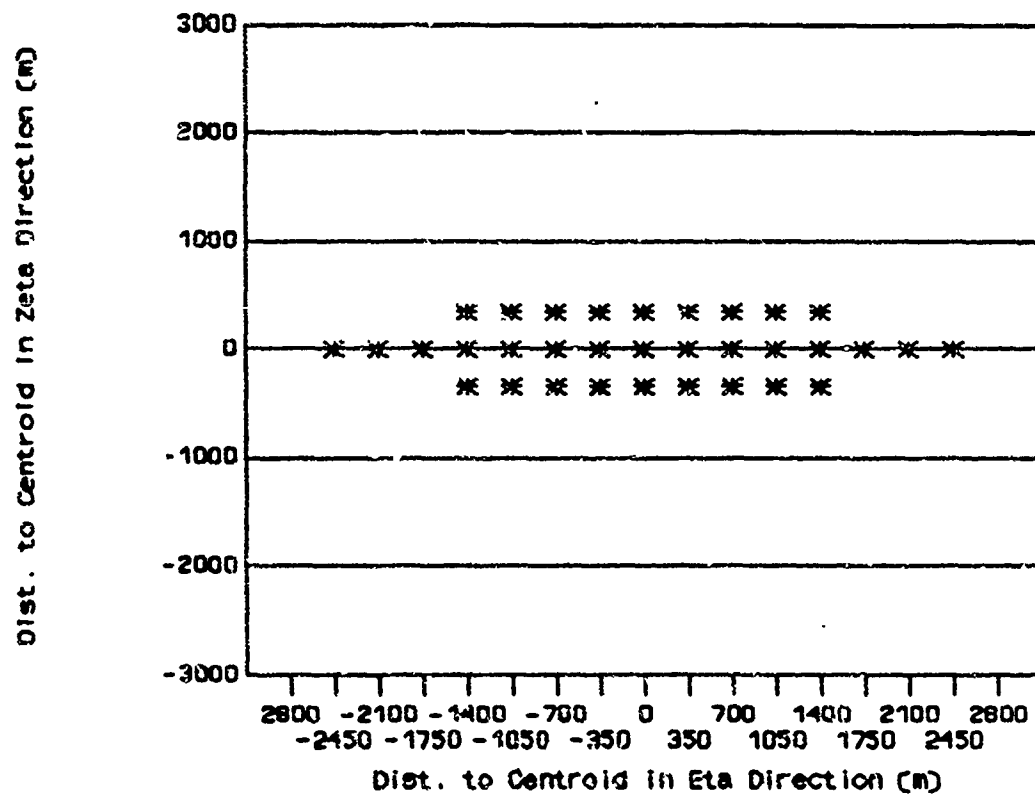
Figure 12. Chaff Cloud Evolution for a 0.01 m/s Dispensing
Velocity: 28800 Seconds After Dispensing
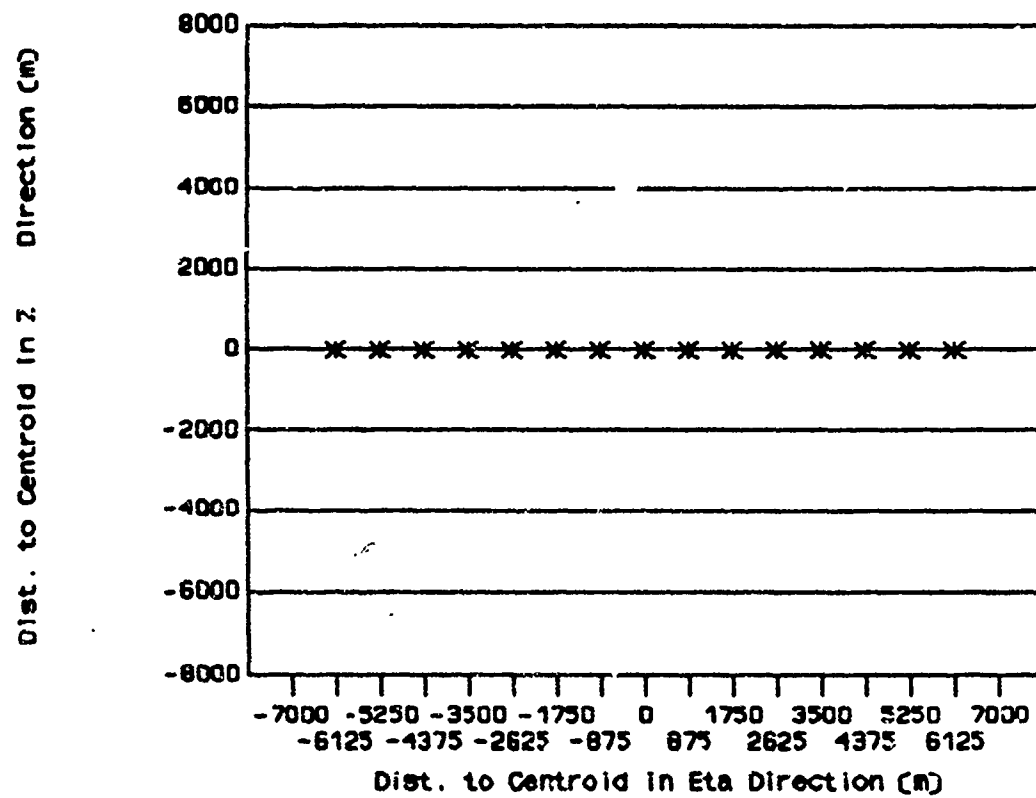
Figure 13.   Chaff Cloud Evolution for a 0.01 m/s Dispensing
Velocity: 39600 Seconds After Dispensing

Although not shown in the figure, the spreading in the ξ direction is very nearly circular. To visualize the expansion of the cloud over time in three-dimensions it is probably easiest to imagine a golf ball, an egg, a cucumber, and then a serving platter. Also not shown in the figure is that the cloud eventually expands to form a toroid around the earth.

As mentioned in Chapter II, those particles dispensed along the path of motion of the parent satellite experience changes in their orbital periods and thus, their velocities. These velocity changes cause the chaff particles to continually spread in the η direction as shown in the figures. Not shown in the figures is the appearance of the cloud in the $\xi\eta$-plane. When viewed from above, the cloud starts out as a circle and then becomes an ellipse. This change in the cloud's shape is caused by the elongation in the η direction.

Also not shown is the displacement in the ξ direction of the "leading" and "trailing" edges of the cloud in relation to its path of motion. In this displacement, the leading edge of the chaff cloud gradually moves farther away from the earth and the trailing edge moves closer. This is caused by the phenomenon noted by Shapiro when he mentioned the "dragon's jaws". Because the eccentricities of the individual chaff particle orbits are disturbed in the

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dispensing process, some of the particles necessarily drift away from the parent satellite's position as the cloud approaches the portion of its orbit 180 degrees from the point of dispensing. If the cloud were examined for a longer period of time, it would continue to oscillate slightly up and down in altitude, but the phenomenon would gradually dampen out.

Finally, the reason the cloud becomes "thinner" as it reaches the 12-hour point of its orbit is simply explained by orbital mechanics. As mentioned in Chapter II, the particles displaced out of the orbital plane will return to the plane of the dispenser every one-half revolution as the two planes cross. At an altitude of 42,073 km, the parent satellite's orbital period is 23.86 hours. Therefore, at very nearly 12 hours the cloud should pinch down somewhat as shown in Figures 10 through 13.

Figure 14 shows a representative graph of the density profile through the central column of the chaff cloud. The density is shown as a function of percentage of distance travelled through the chaff cloud, with the density being greatest in the center of the cloud. The graph presented is for a dispensing velocity of 0.05 m/s and at a time after dispensing of five hours. Other combinations of dispensing velocity and time after deployment have similar density profiles with profiles of the cloud at earlier times having

78
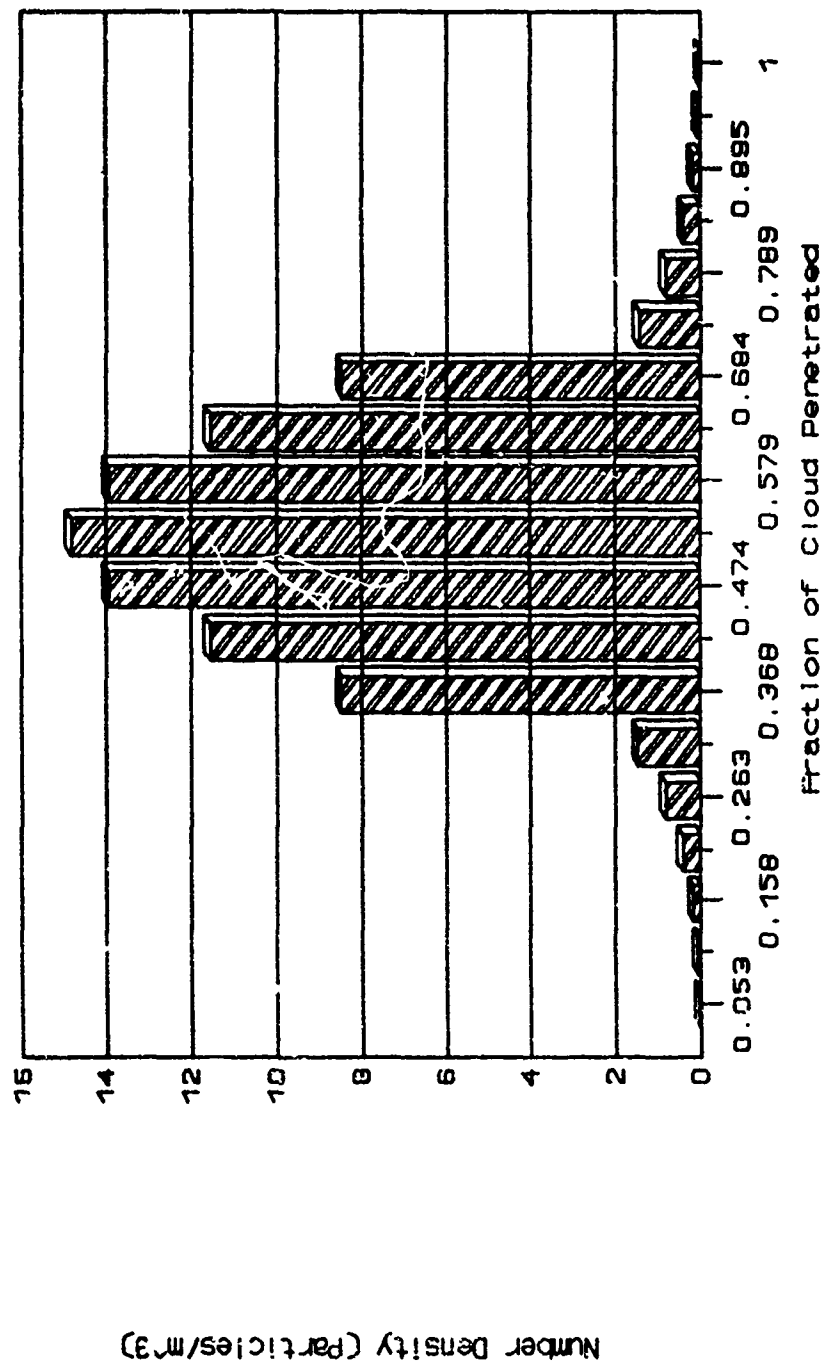
Figure 14. Average Density Through the Centroid of the Chaff Cloud

a higher density at the center and a lower density at the outer edges and with the opposite being true for later times after deployment. For smaller dispensing velocities, the central densities at the same point in time would be higher; for higher velocities they would be lower.

Signal_Attenuation_After_One_Hour

This section discusses the amount of signal attenuation affected by the chaff cloud one hour after initial dispensing. Three areas are addressed for various dispensing velocities: attenuation of the signal exactly one hour after dispensing, attenuation of the signal in ten minute increments leading up to one hour after dispensing, and the time it takes for the chaff cloud to achieve a -10 dB attenuation.

Attenuation_a*_One_Hour. This section answers the question of whether the chaff cloud can affect at least a -10 dB attenuation of the input signal at a time of one hour after initial dispensing. Since dispensing can occur at virtually any velocity, attenuation is analyzed for dispensing velocities between 0.005 m/s and 0.05 m/s. The reason for limiting the upper velocity at 0.05 m/s will be explained later in the chapter.

Figure 15 shows the attenuation in dB of the signal after it passes through the chaff cloud at a time of one hour for the dispensing velocities of 0.0115 to 0.0155 m/s. As would be expected, the attenuation is greater for the higher dispensing velocities than for the lower velocities. This is because the faster dispensing causes the particles to expand and fill the required cross-sectional area faster than the slower velocities. With more particles within the outer region of the central column, more attenuation is affected. Also, the greater velocity has the effect of making the thickness of the central column increase. With more distance for the signal to traverse, more attenuation is affected.

From the figure, it can also be seen that the attenuation is less than -10 dB for velocities less than approximately 0.014 m/s. Therefore, the dispensing velocity has to be greater than this value for the chaff cloud to be effective.

The range of dispensing velocities examined for this study was expanded in Figure 16 so velocities between 0 and 0.05 m/s could be examined for signal attenuation at a time of one hour after dispensing. As before, the attenuation generally increases with increasing dispensing velocities. However, at a velocity of approximately 0.04 m/s the attenuation level begins to decrease. There are two

81
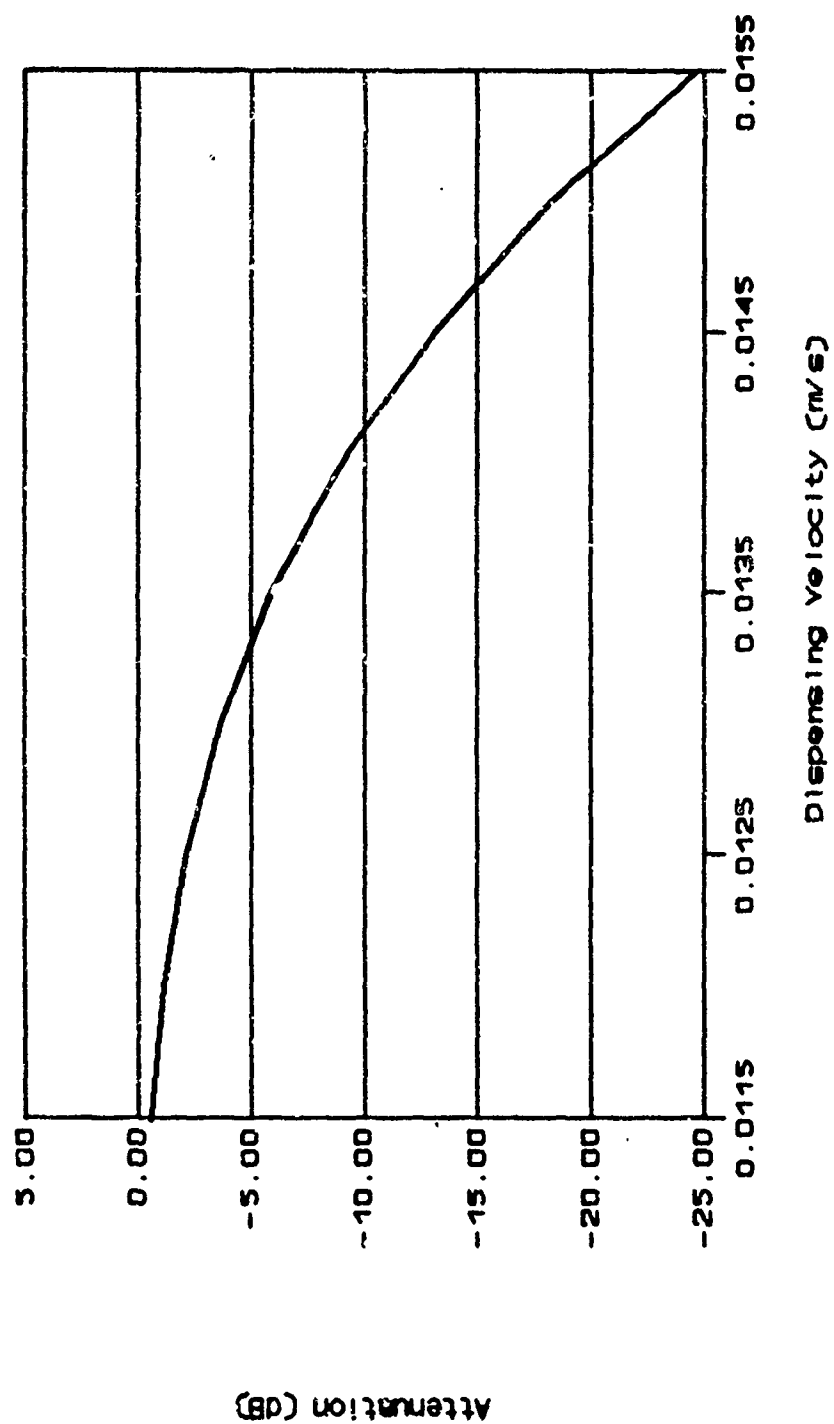
Figure 15. Signal Attenuation at One Hour for Dispensing Velocities
Between 0.0115 and 0.0155 m/s
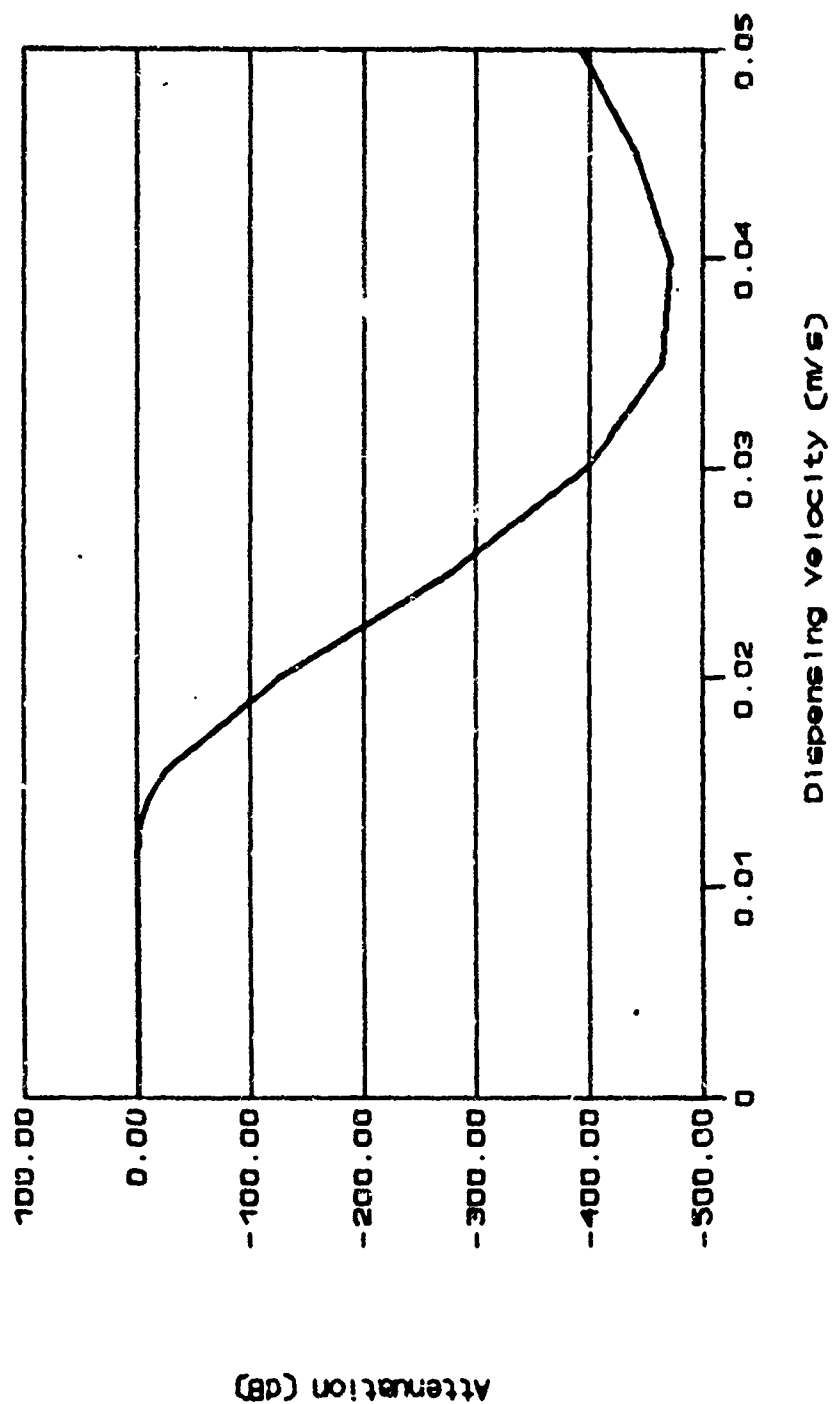
Figure 16. Signal Attenuation at One Hour for Dispensing Velocities Between 0 and 0.05 m/s

phenomena which contribute to the increasing-decreasing attenuation: a decreasing number of particles in the centroid of the chaff cloud and an increasing path length through the chaff cloud.

For velocities less than 0.04 m/s, not enough particles are able to reach the outer edge of the central column to affect a -10 dB attenuation within one hour. For velocities above 0.04 m/s, particles are retreating from the centroid at a rate fast enough that the number left within the centroid decreases below that required for the greatest attenuation. In fact, the maximum attenuation at one hour for the central subvolume occurs for a velocity of about 0.035 m/s. The increasing path length now becomes important to the attenuation of the signal. At dispensing velocities less than 0.04 m/s, the chaff cloud attenuation at the one hour point is accomplished almost entirely by the particles within the subvolume at the centroid of the cloud. At higher velocities, a greater percentage of the signal attenuation is caused by the particles that lie in subvolumes farther from the centroid. It is this combination of attenuation due to the large number of particles in the centroid and attenuation due to the increasing number of particles along the ever increasing path length that cause 0.04 m/s to be the velocity with the highest attenuation. For velocities above 0.04 m/s, the

attenuation in the centroid drops off faster than the surrounding subvolumes can compensate, so the overall attenuation for the chaff cloud gradually begins to decrease.

Attenuation_in_Ten_Minute_Increments.   Figure 17 shows plots of attenuation versus dispensing velocity for times after dispensing of 10, 20, 30, 40, 50, and 60 minutes. Again as expected, the attenuation is greatest for the highest dispensing velocities for each point in time. From the figure it is clear that at times of 10 and 20 minutes after dispensing, almost none of the dispensing velocities acheive an effective level of attenuation. However, for times greater than 30 minutes after dispensing, the attenuation levels acheived are fairly significant for all dispensing velocities above 0.03 m/s. The decrease in attenuation found in Figure 16 is also apparent in Figure 17.

Time_Until_-10_dB_Attenuation. Although not specifically outlined as a requirement by Brown, it would be nice to know the time it takes for the chaff cloud to acheive a -10 dB attenuation for various dispensing velocities. Figure 18 shows, to an accuracy of plus or minus five minutes, the time it takes the chaff cloud to attain an attenuation level of at least -10 dB. For velocities of 0.03 m/s, effective attenuation is achieved within the first 30 minutes or in

85
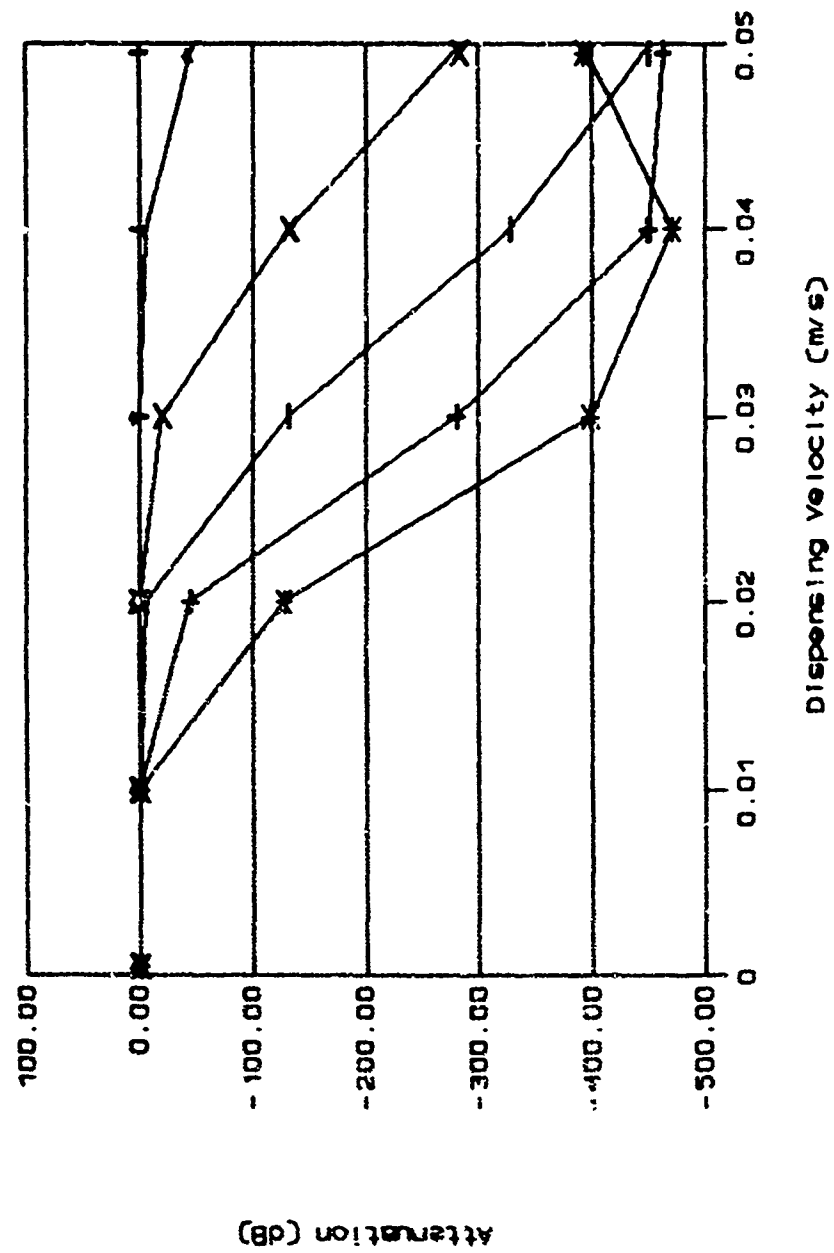
Figure 17. Signal Attenuation in Ten Minute Increments Up to One Hour

86
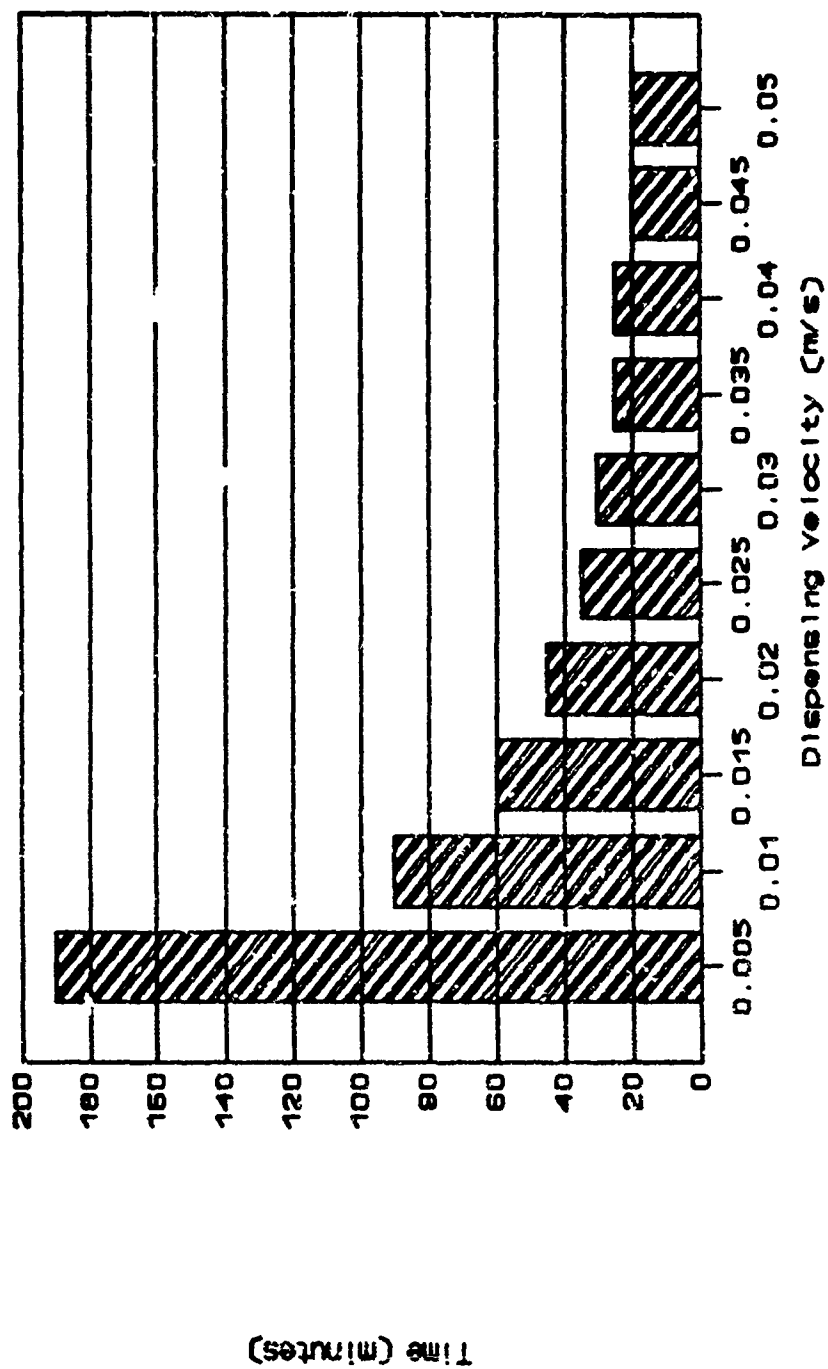
Figure 18.   Time Until -10 dB Attenuation

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half of the required time. However, for velocities of 0.01 and 0.005 m/s the time before effective attenuation is achieved are 90 and 190 minutes, respectively.

The reason for the great disparity in the time until effective attenuation occurs can be explained by looking at the Maxwellian velocity distribution used for the chaff cloud. The distribution has a negative exponential function which decreases to zero for increasingly large exponents. However, the dispensing velocities are placed in the denominator of the exponent so larger velocities cause the function to increase rather than decrease. The additional effect of squaring the dispensing velocities in the equation causes small changes to have a relatively large effect.

The overall effect of the Maxwellian velocity distribution is that most of the particles will have velocities near the dispensing velocity with a few with much greater velocities as the function decreases in the tail region. Therefore, for a dispensing velocity of 0.005 m/s, not many particles will have velocities even as high as 0.05 m/s and only a very small number will have higher velocities. However, for 0.05 m/s, most particles will have a dispensing velocity of 0.05 m/s with a few having much greater velocities. The result is that the area required of the chaff cloud for attenuation of a signal is covered by

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particles at a much faster rate for larger dispensing
velocities than for lesser velocities.

Attenuation The First Twelve Hours After Dispensing

This section examines the attenuation affected by the
chaff cloud for the first twelve hours after dispensing,
since the second requirement for attenuation is that the
cloud maintain an attenuation of -10 dB for at least twelve
hours. The twelve hour attenuation is examined in two
parts. In the first, the attenuation of dispensing
velocities between 0.01 m/s and 0.05 m/s will be examined by
looking at individual plots of attenuation. In the second,
the attenuation levels are examined on a collective plot
where all the velocities are represented.

Individual Dispensing Velocities for Twelve Hours.
Figures 19 through 23 show plots of signal attenuation
versus time for dispensing velocities of 0.01, 0.02, 0.03,
0.04, and 0.05 m/s respectively. Figure 19 shows the signal
attenuation increasing up to the four-hour point at which
time it begins to decrease. Figures 20 through 23 show
similar occurences with the maximums being reached at times
of two hours, one hour, one hour, and 50 minutes,
respectively. These decreases in attenuation are caused by
the same phenomena mentioned in the previous section where

89
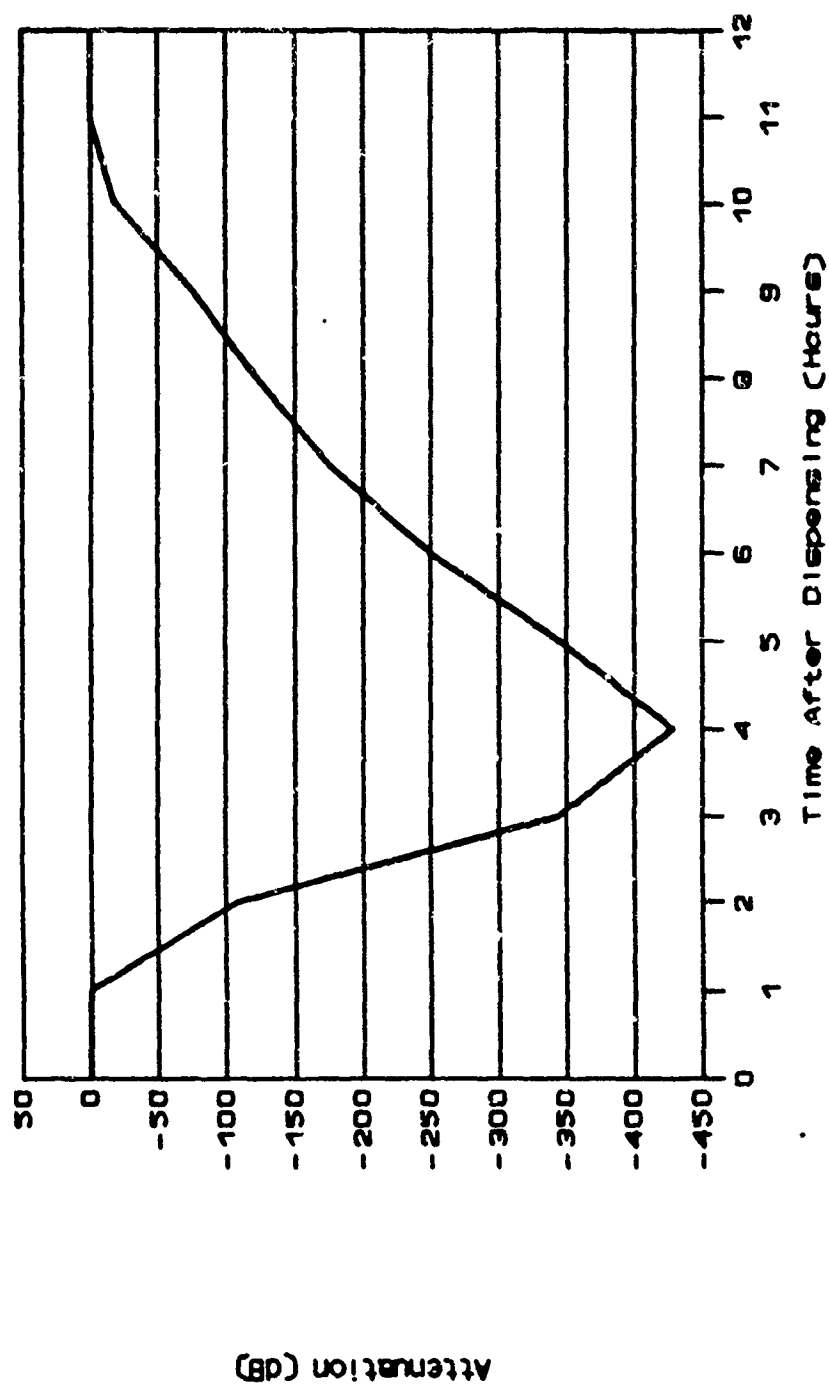
Figure 19. Attenuation for the First Twelve Hours: 0.01 m/s Dispensing Velocity

90
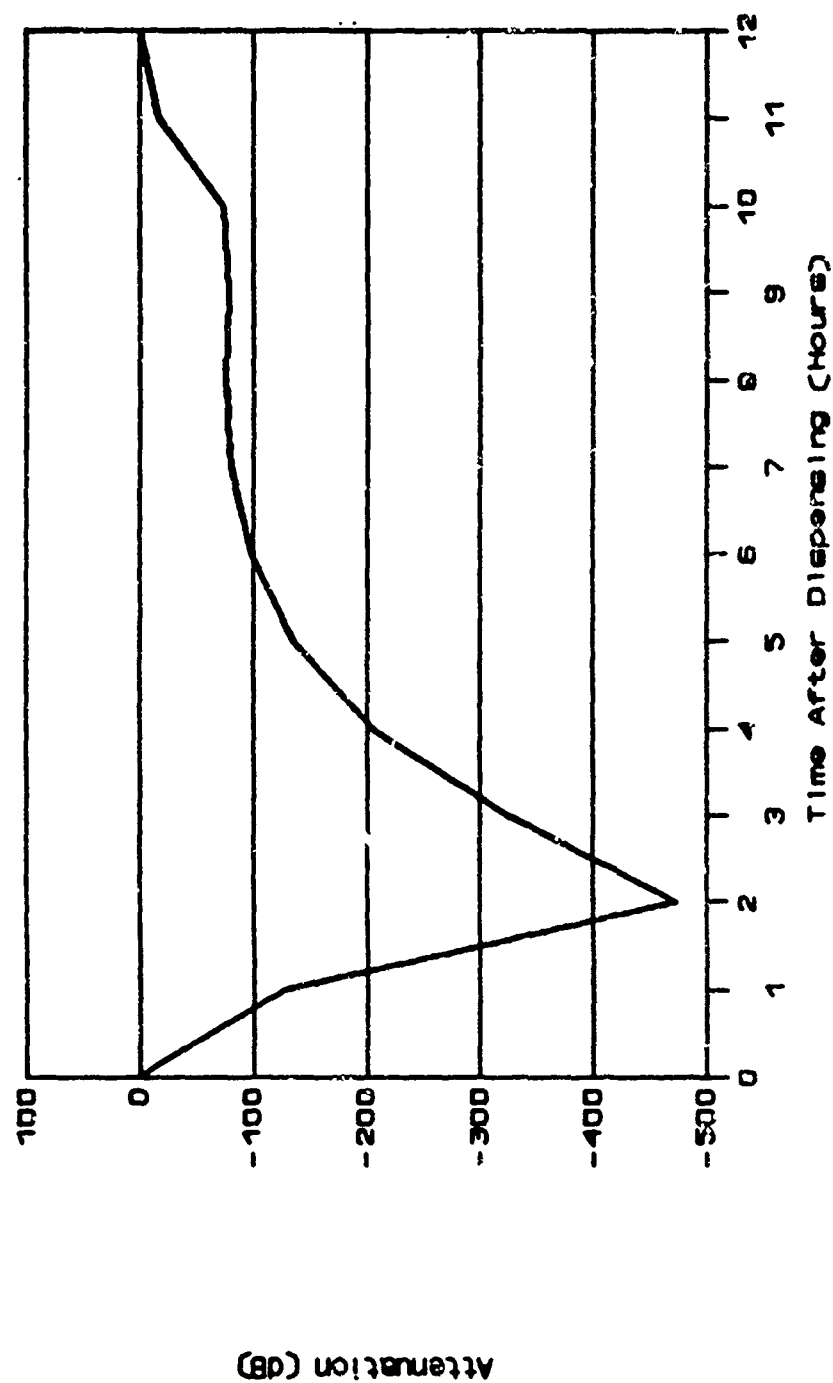
Figure 20. Attenuation for the First Twelve Hours: 0.02 m/s Dispensing Velocity

91
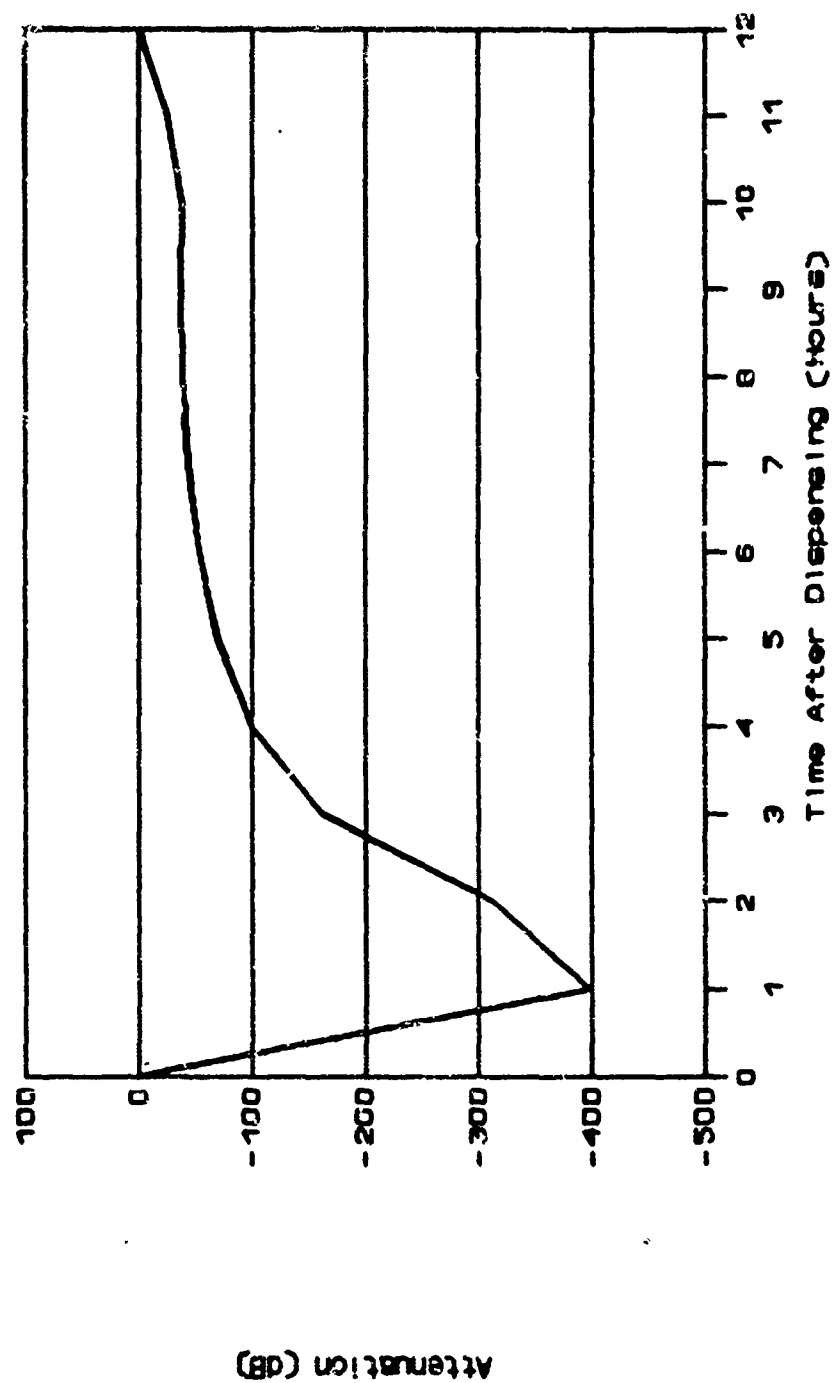
Figure 21. Attenuation for the First Twelve Hours:
0.03 m/s Dispensing Velocity
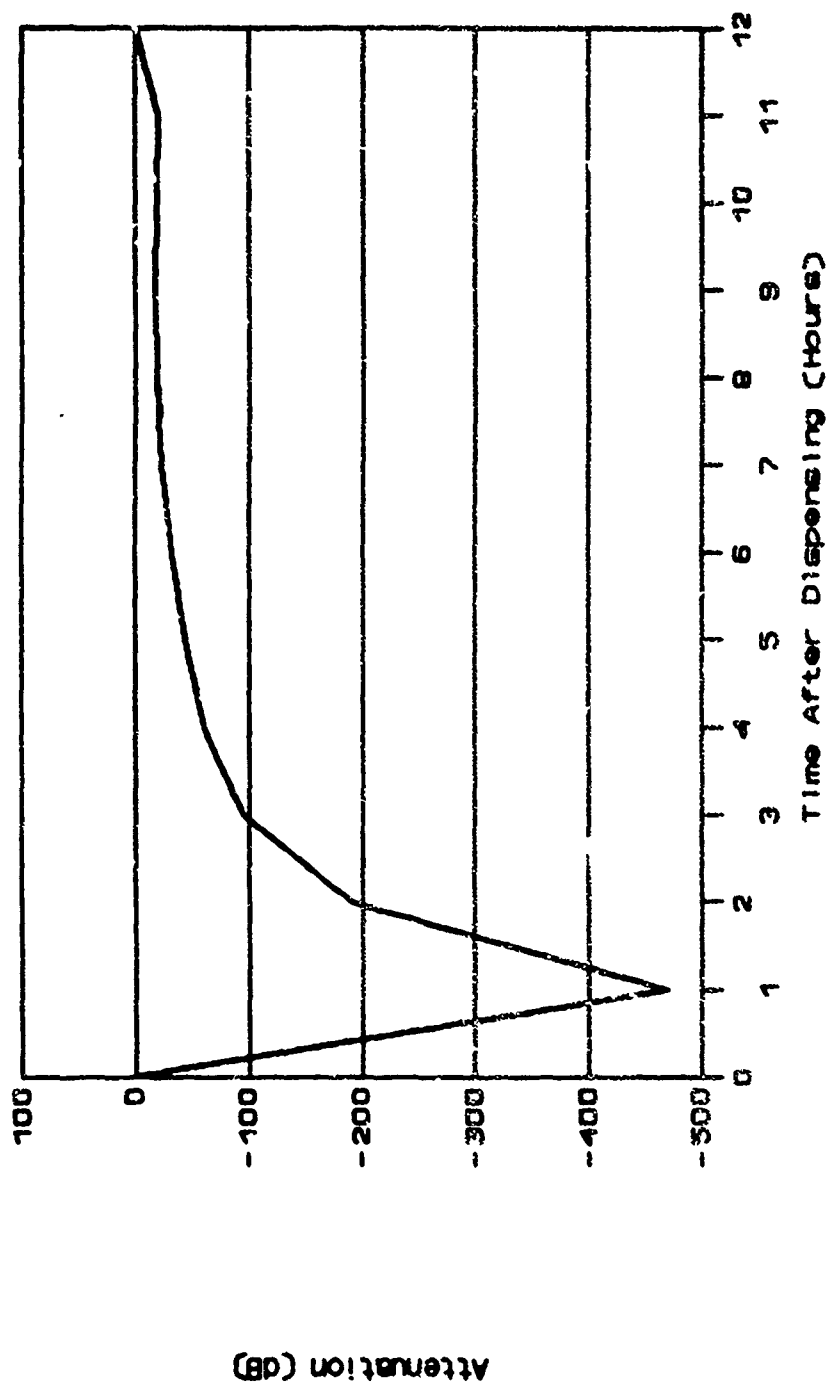
Figure 22. Attenuation for the First Twelve Hours:
0.04 m/s Dispensing Velocity

93
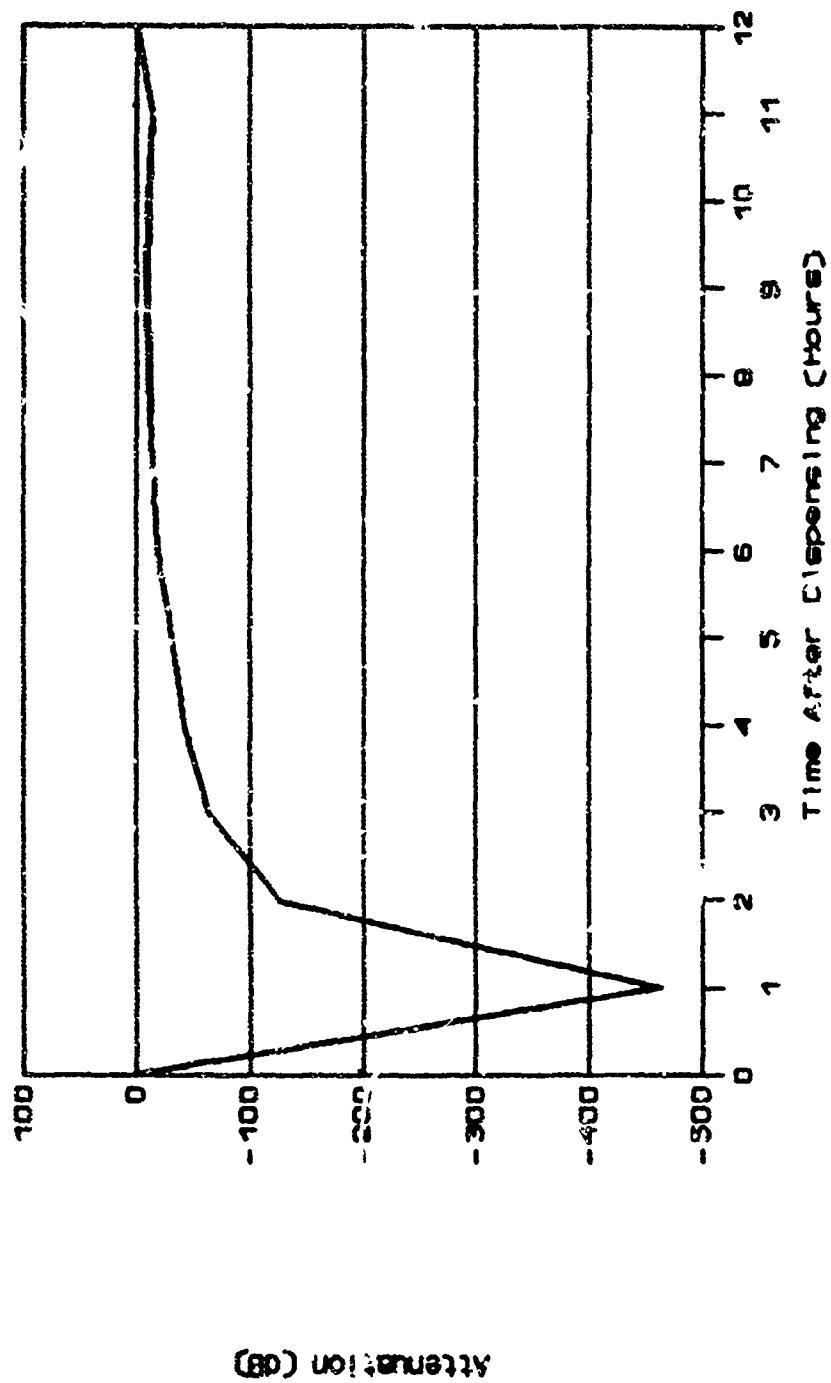
Figure 23. Attenuation for the First Twelve Hours
0.05 m/s Dispensing Velocity

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the number of particles left in the centroid of each cloud drops below the number required for maximum attenuation.

Another feature in each of the figures is that the attenuation reaches its maximum value at a faster rate than the attenuation decreases beyond the maximum. Thus, each of the plots take on the shape of "V" with the left leg at a greater angle with respect to the horizontal than the right leg. This feature is most prominent for the higher velocities.

All figures also show the attenuation levels dropping below the -10 dB point before the twelve-hour point is reached. Therefore, none of the clouds meet the second attenuation requirement set by Brown. This problem can be explained by examining the orbital mechanics of the cloud. As mentioned previously, the cloud pinches down as it approaches the twelve-hour point, causing the cloud to become smaller in the ξ direction than that required for proper attenuation across the required cross-sectional area. The only way to get the cloud to pinch down after 12 hours would be to deploy it far enough away from the earth so that its orbital period would be greater than 24 hours. However, since this would cause the cloud to lie outside the orbit of the DSCS satellite it is attempting to jam, the cloud would be rendered useless as a jammer.

Examination of the figures also shows the following
interesting feature for all but the 0.01 m/s dispensing
velocity: As the time approaches the twelve hour point, the
attenuation (which was decreasing) begins to increase again.
This can be explained by the fact that the cloud is
expanding along the central column in the ξ direction. When
a cloud becomes large enough, the fact that attenuation
through any one subvolume along the central column does not
greatly attenuate the signal is counteracted by the
multiplicative effect of the signal passing through more and
more subvolumes along the way. For example, if each of 21
subvolumes along the signal's path cut the signal by only 15
percent, the final output signal would be 0.032 of the input
signal. This equates to an attenuation of -14.8 dB. The
fact that the attenuation again drops off after this portion
of the curve for each figure is due to the pinching down of
the cloud which was described above.

For most of the dispensing velocities examined, once
attenuation of greater than -10 dB is attained, it remains
above that level until the pinching of the cloud causes
attenuation to decrease again. However, the level of
attenuation at which the attenuation stops decreasing and
begins to increase again becomes lower for increasing
velocities. For example, the attenuation of the 0.02 m/s
dispensing velocity cloud began to increase again after

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attenuation dropped to about -75 dB. But, for a dispensing velocity of 0.05 m/s, the attenuation did not increase until it was slightly below -10 dB. Therefore, for dispensing velocities greater than 0.05 m/s, the attenuation level would also drop below -10 dB before the attenuation level began to increase again. Since the attenuation of the cloud with a dispensing velocity of 0.05 m/s dropped below -10 dB after only nine hours, higher dispensing velocities would attenuate for even shorter periods of time and, thus, were not examined.

Collective_Examination_of_Dispensing_Velocities_for Twelve_Hours_. When Figures 19 through 23 are superimposed upon each other, Figure 24 results. Figure 24 looks pretty much as expected, with the curves for lower dispensing velocities lying below and the to the right of the higher dispensing velocities. However, a closer examination shows both the 0.01 and 0.02 m/s curves crossing the other curves beginning at about nine and eleven hours, respectively.

The reason behind this apparent discrepancy with the previous results is once more due to the interaction of the three major functions affecting attenuation: the number of particles within the subvolumes, the size of the chaff cloud, and the pinching of the cloud at the twelve hour point. Since the chaff clouds with higher dispensing

97
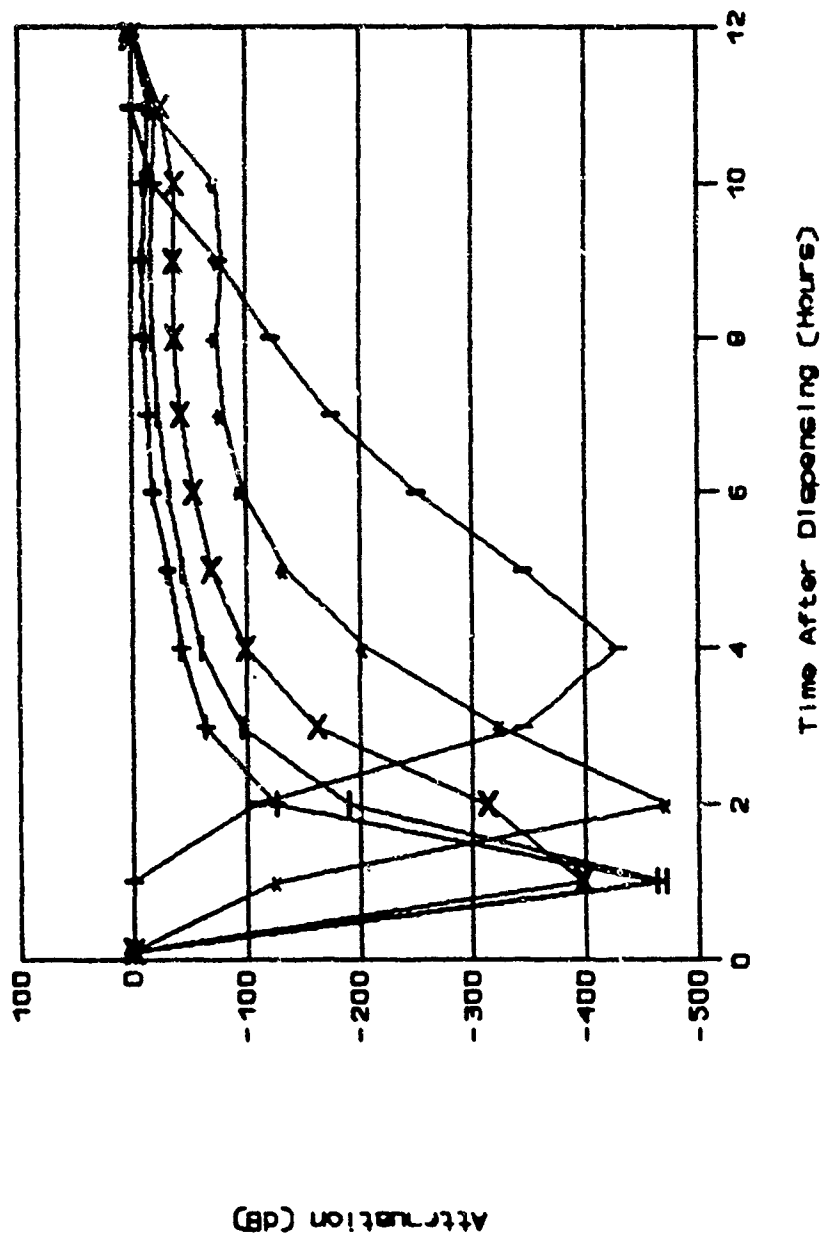
Figure 24. Attenuation for the First Twelve Hours: 0.01, 0.02, 0.03, 0.04, and 0.05 m/s Dispensing Velocities

velocities expand much more rapidly than the clouds with
smaller dispensing velocities, the multiplicative effect of
the attenuation through the subvolumes is greater for them
than for the clouds with slower dispensing velocities.
Thus, despite the fact that the clouds with slower
dispensing have more particles contained near their
centroids than the clouds with faster dispensing, the small
number of subvolumes along the central column becomes an
inhibiting factor to overall signal attenuation.

For example, at ten hours, the attenuation through the
centroid of the cloud with a 0.01 m/s dispensing velocity is
97.4 percent, while it is only 86.4 percent for the cloud
with a 0.05 m/s dispensing velocity. However, with 11
subvolumes 0.35 km on a side along the central column, the
cloud with the higher dispensing velocity has an overall
attenuation of 99.98 percent. The cloud with a 0.01 m/s
dispensing velocity has only five subvolumes along the
central column for an attenuation of 98.8 percent. The
reason the clouds with 0.03 and 0.04 m/s dispensing
velocities do not cross over the other curves is because the
pinching effect of the chaff cloud begins to dominate the
expansion function for these velocities before the expansion
functions show up on the graphs.

VI. Conclusions and Recommendations

This chapter summarizes the research done to study the
deployment of a chaff cloud in space. First, it lists and
discusses conclusions that can be drawn directly from the
results. Then it lists some recommendations for areas of
further study and briefly discusses these recommendations.

Conclusions

The results of the previous chapter clearly demonstrate
that no dispensing velocity for the chaff cloud allows it to
attenuate the incident communications signal for twelve
hours because of the laws of orbital mechanics. Also
demonstrated was that dispensing velocities less than
0.014 m/s or greater than 0.05 m/s are inadequate, in the
first case because the signal is not attenuated quickly
enough and in the second case because signal attenuation
drops below -10 dB at a time of nine hours after dispensing.

Since none of the dispensing velocities attenuate for a
full twelve hours, a compromise with the attenuation
requirements set by Brown will have to be reached. It is
probably best to choose a dispensing velocity for the chaff
cloud that causes attenuation as quickly as possible and
that also continues to attenuate for as long as possible.

The early attenuation will help counteract the fact that
attenuation does not continue up to the twelve hour point.
Dispensing velocities greater than 0.03 m/s seem to be best
since they can attenuate the signal within 30 minutes and
continue to attenuate the signal up to the point the cloud
pinches down near twelve hours. However, the attenuation by
the clouds with dispensing velocities of 0.04 and 0.05 m/s
drop as low as -17 and -9.8 dB well before the twelve hour
point, while the attenuation by the 0.03 m/s velocity cloud
drops only as low as -37 dB. For this reason, the
dispensing velocity of a chaff cloud in space should be
approximately 0.03 m/s to best acheive the attenuation
requirements suggested by Brown.

Recommendations

With the feasibility of using a chaff cloud in space as
a communications jamming device demonstrated, it is now
necessary to ask a few questions regarding the assumptions
used in the study and to outline some areas of further
research concerning the chaff cloud. This section asks the
following four questions:

> 1) Are the attenuation levels acheived by the
>   chaff cloud high enough to make the cloud
>   useful?

2) How does relative motion between the centroid
of the chaff cloud and the target satellite
affect attenuation?

3) Are the assumptions regarding the dispensing
of the chaff cloud reasonable?, and

4) What are the effects of the chaff cloud on
other satellites?

Is_the_Chaff_Cloud_Useful? Although the chaff cloud

does not maintain a significant level of attenuation for a

full twelve hours, the fact that it does maintain levels of

more than -50 dB for several hours is certainly significant.

Therefore, it would be worthwhile to look into the

significance to communications of the levels of attenuation

attainable by a space-borne chaff cloud. Areas addressed

could include: the length of time attenuation needs to be

maintained in order for communications to be sigificantly

disrupted; levels of attenuation required to disrupt most

satellite communications, and what conditions apply to each

level; and what sort of tradeoffs can be made between

attenuation levels achieved by a chaff cloud and the periods

of time over which the levels are maintained so that the

most significant signal disruption occurs. All these

questions could be addressed to the various dispensing

velocities available for the chaff cloud to help redefine

limits for the model.

Another question that needs to be asked is what happens to the signal attenuation after the twelve hour point? Since the cloud will begin to expand again after the twelve hour pinching, perhaps a few more hours of attenuation can be acheived by the cloud after attenuation levels build back up again. The levels of this second period of attenuation need to be examined to see how long they remain significant if, in fact, they are. Maybe the pinching down can be counteracted by another attenuation device until the cloud expands again. Finally, since the chaff cloud is continually expanding and contracting causing periodic signal disturbances, perhaps nothing more needs to be done to significantly disrupt communications.

What_is_the_Effect_of_Motion_Between_the_Chaff_Cloud and_the_Satellite? As Brown points out in his thesis, the chaff cloud has a motion relative to a geosynchronous satellite of 7 m/s. This is due to the fact that the cloud is in a lower orbit around the earth than the target satellite. Since the chaff cloud is expanding at a much slower rate than this relative motion, the chaff cloud will move out of a position of effective attenuation well before maximum attenuation is reached (Brown, 1987:4-18 to 4-19).

There are several ways of dealing with this problem: deploying the cloud closer to a geosynchronous altitude to

minimize the relative motion; deploying the cloud earlier in time so it drifts across the geosynchronous satellite's field of view after the cloud has grown large enough to effectively attenuate the signal; deploying the chaff cloud from an elliptical orbit and allowing the changing velocity of the chaff cloud, as it follows the elliptical path, to keep the cloud in the geosynchronous satellite's field of view longer; and deploying more than one chaff cloud in orbit, and allowing them to drift past the geosynchronous target in succession.

By deploying the chaff cloud from a higher altitude, the relative motion between it and a geosynchronous target can be reduced substantially. However, if the relative motion was to be reduced to 0.5 m/s, the chaff cloud would be deployed only 13.8 km below geosynchronous altitude. In this case, the relative motion between the two bodies would still be too high as it would take a 2.45 km diameter chaff cloud only a little less than an hour and a half to pass completely in front of a geosynchronous satellite. More importantly, the chaff cloud would be unacceptably close to the target satellite, as expansion of the chaff cloud would eventually cause it to collide with the satellite.

Perhaps the chaff cloud can be dispensed before it is actually needed so that its expansion would be big enough to cause effective attenuation over the required area on the

ground as it drifted past the geosynchronous satellite. This would eliminate the need to wait for the cloud to expand as it drifted out of view of the target satellite. If the timing and direction of motion of the chaff cloud was right, attenuation levels of several hours might be realized, since the chaff cloud expands fastest along its path of motion. For example, the cloud could be dispensed so that just as it reached a length of 2.45 km in the ζ direction, it would begin to cross the path of the uplink signal to the target satellite. By the time the cloud drifted past the geosynchronous satellite, several minutes would have elapsed. This option might also work with the cloud at a higher altitude so the relative velocity between the two bodies is minimized. Both of these options should also be examined in connection with the level of attenuation/time of attenuation calculations mentioned previously, in order to best meet attenuation requirements.

The third option, deploying the chaff cloud in a slightly elliptical orbit such that it slows down as it passes the geosynchronous satellite, was posed by Brown in his thesis. The idea is based on the fact that a satellite travels at a slower velocity at apogee than at perigee. Brown showed that the chaff cloud can be deployed in an orbit with properly chosen apogee and perigee altitudes so that the centroid of the chaff cloud will remain within 42

meters of a point located directly in front of the satellite
for twelve hours. This occurs because the satellite catches
up to and slightly passes the centroid of the chaff cloud as
the cloud approaches its apogee. The satellite then slowly
falls behind the chaff cloud as the cloud speeds up on its
way to perigee. According to Brown, the orbit for which
this will work has a perigee 100 NM below geosynchronous
altitude and an apogee at geosynchronous altitude (Brown,
1987:4-19 to 4-22). However, Brown neglects to examine the
pinching down of the chaff cloud or the possibility of
collisions between the cloud and the geosynchronous
satellite in his analysis, so further study in this area is
warranted.

Finally, the chaff cloud could be dispensed as a series
of clouds along the path of relative motion which exists
between the geosynchronous satellite and the chaff cloud.
This way, as one cloud passes the field of view of the
geosynchronous satellite, another would appear in its place.
If enough chaff clouds were dispensed in this manner, the
relative motion might no longer pose a problem. Perhaps
even the problem of the cloud pinching down could be
eliminated by spacing the clouds so that as one cloud began
pinching down, another would expand in its place.

Are the Dispensing Assumptions Reasonable? Many
assumptions went into the chaff cloud deployment model and
while all are reasonable for the first-order calculations of
this study, further analyses would require a more critical
look in the following areas: weight and volume restrictions
on a space-deployed chaff cloud, ability to meet dispensing
velocity requirements with a spherical cloud deployment,
possibility of birdnesting of chaff particles, and actual
velocity distribution of chaff particles.

With 100 billion chaff particles being deployed, the
weight and size of the chaff package to be put near
geosynchronous altitude could be significant. According to
Brown, the volume of a single chaff particle of the size
used for this study is 1.47×10^{-12} m^3. The particle density
and appropriate packing density for the chaff package are
2550 kg/m^3 and .55, respectively.(Brown, 1987:4-5).
Therefore, the particles weigh 375 kg (826 lb) and occupy
2.61×10^{-1} m^3. The volume occupied by the chaff particles
is fairly small so the chaff package could probably be
easily launched into orbit as a "piggyback" on another
satellite. However, the weight is high and would probably
require a separate launch to reach orbit. After launch
costs are considered, one chaff cloud could easily cost
hundreds of thousands of dollars to deploy. If higher
attenuation values were desired, the cloud would weigh and

cost even more. One possibility would be to reduce the number of chaff particles. This, of course, would require another attenuation study, but a factor of 10 reduction in the number of particles would result in a more manageable weight of 37.5 kg. If the reduction in attenuation were not too great, the deployment of the cloud might be feasible.

The second dispensing assumption that needs further consideration is the initial spherical dispensing pattern. Although an exploding ball of chaff with a central charge could be constructed, it is not clear if dispensing velocities of less than 0.05 m/s could also be realized. If not, another dispensing mechanism needs to be devised. Additionally, Brown suggests that an explosive charge might damage a large number of the chaff particles (Brown, 1987:4-11). Brown also suggests dispensing the chaff particles using spools of chaff from which the chaff is deployed as the spools rotate. This is how the dipoles were dispensed for Project West Ford. For West Ford, the dipoles were attached to the spools with napthalene which melted upon exposure to sunlight, thus releasing the dipoles over time. All the spools were initially spun about the axis of minimum moment of inertia so that the release of the dipoles would upset this inherently unstable condition and cause the spools to tumble. Thus, the dipoles were released in all directions (Brown, 1987:4-11 to 4-12). By dispensing the

particles in this manner, however; the model used in this study would be invalidated, since the model used assumed instantaneous dispensing. Maybe a compromise could be reached between dispensing techniques, or Heard's long-term dispensing model could be applied.

Another problem with dispensing chaff particles in space is "birdnesting", the clumping together of particles due to attractive electrical charges on the particles. According to MacLellan, up to 50 percent of the dipoles for project West Ford were involved in birdnesting (MacLellan, 1964:564). This condition would be expected with the dispensing of any lightweight metallic objects in space since there are no aerodynamic forces present to help the particles separate upon deployment. Since all particles in the model were assumed to properly deploy, a correction factor might need to be used to take birdnesting into account, but proper values for the number of particles involved need to be established first.

The final assumption associated with the dispensing process that deserves additional consideration is the velocity distribution function. Although a Maxwellian distribution seems appropriate for the chaff cloud in general, the Maxwellian distribution technically has no upper limit on particle velocity while the chaff particles

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definitely have an upper limit on their velocity. This
problem becomes important for the chaff cloud because of the
large number of particles involved. If only 0.001 percent
of the particles had a velocity above a certain value, the
number of chaff particles involved would be 1 million.
Thus, the model would show a large number of particles
located a much greater distance from the centroid of the
chaff cloud than there really were. This discrepancy could
greatly affect the attenuation calculations, depending on
which portion of cloud was being considered and how much
time had elapsed since dispensing. Therefore, a new model
which properly places an upper bound on dispensing velocity
should be looked into if chaff deployment in space is to be
further considered.

   What_Are_the_Effects_of_the_Chaff_on_Other_Satellites?
Chaff can be expected to have two possible interactions with
satellites: attenuation of communication signals and
collisions.

   Since the chaff cloud is moving with respect to the
satellite it is attempting to jam, it has the possibility of
drifting in front of other satellites and disrupting their
communications. In order to fully assess the impact on
communications, the locations and transmitting/receiving
frequencies of other nearby satellites would have to be

examined before each chaff cloud deployment. Also, if the
frequency the chaff was designed to attenuate was not the
same one used by other satellites in the vicinity, even if
the chaff cloud passed directly in front of the other
satellites, only noise would result. Therefore, even if the
chaff cloud did accidentally get in between a satellite and
its ground station, the signal might not be attenuated below
the level necessary for uninterrupted comunication.
However, an increase in noise for all satellites the chaff
cloud passes in front of can be expected. On the plus side,
perhaps the cloud could be deployed to jam more than one
satellite for specified periods of time if the satellites
were close enough together. Further research into these
areas is clearly needed.

Also of major concern should be the possibility of
collisions between the chaff particles and other satellites
in earth orbit. Fortunately, the chaff cloud as envisioned
is well below the altitude of geosynchronous satellites, so
impacts with them would not be a problem. However, solar
radiation pressure and other perturbative forces will cause
the chaff particles to decay into lower orbits, so they
could eventually come in contact with satellites nearer to
the earth.

Since the chaff particles are so tiny, they at first
would not seem to pose much of a threat. However, with the

chaff cloud moving at about 3 km/s, impacts could be devastating. According to Donald J. Kessler, a NASA scientist who studies space debris, objects between 1 mm and 1 cm in size can penetrate spacecraft walls if they are moving at velocities of 10 km/s (Kessler, 1987:587). With the chaff particles being 1.87 cm long (Brown, 1987:4-3), although also being very thin and moving much slower than 10 km/s, impacts might still be a problem.

Even if the chaff particles do not damage the satellites they come into contact with, they could interfere with the satellites' operating functions by sticking to the satellites' surfaces. Since the chaff particles are at least partially composed of metal, the bombardment of the chaff particles by high energy particles from the sun will likely cause the chaff particles to acquire an electric charge. If the charged chaff particles were to fly within close proximity of an oppositely charged satellite surface, they would be drawn to the satellite and stick to the surface. If enough particles stuck to solar panels or antennas, a degradation of satellite operations might result. Therefore, this is another area which warrants further study.

Two suggestions were proposed by Brown to help eliminate the problem of unwanted signal attenuation by and impacts with the chaff cloud: constructing the chaff of

materials so fine they would be broken up and swept away by the solar wind, or using materials for the chaff that deteriorate when exposed to ultraviolet light (Brown, 1987:4-23 to 4-24). Although Brown did not go into much detail on these suggestions, they might deserve further consideration if a chaff cloud is to be deployed in space.

Appendix A: Density Spreadsheet Formulas

This appendix contains a printout of all the formulas used in the density spreadsheet. Each cell of the spreadsheet is listed on a separate line. The cell labels are set up so that letters represent columns and numbers represent rows starting from the upper left corner of the spreadsheet. Therefore, entry "D10" is the fourth entry from the left in the tenth row of the spreadsheet.

In the formula printout, several symbols are used in front of the cell entries. An apostrophe, a set of quotation marks, or a caret symbol (^) indicate that the entry in the cell is a label and not used in any calculations. The symbol "[w10]" is used to represent each new row of the spreadsheet, and the symbol "\-" indicates that the cell is filled with a row of hyphens. All cells without one of these marks are either blank or contain formulas.

Rows 1 through 20 contain constants and variables for use in calculations by the rest of the spreadsheet. The constants are those used by Heard in Eq (40) and the variables are the following: the spreadsheet scaling factor, the square of the thermal velocity of the chaff particles, the time after dispensing, the number of chaff

particles in the spreadsheet grid, the dispensing velocity of the chaff particles, and the displacement coordinates of the centroid of the spreadsheet grid from the centroid of the chaff cloud.

Rows 21 through 78, 80 through 137, 139 through 196, 198 through 255, 257 through 314, 316 through 373, and 375 through 430 calculate the relative number densities for each of the seven values of ξ contained in the spreadsheet model grid. Within each of these sections are subsections which calculate values for P1, P2, and P3; a logic table to help the spreadsheet calculate exponents of negative numbers smaller than negative 200; and calculations of the relative particle densities as solutions for Eq (40).

Finally, Row 433 calculates the sum of the relative number densities and rows 436 through 509 calculate the actual number of particles for each (ξ, η, ζ) coordinate within the 7x7x7 grid modeled by the spreadsheet.

```
A1:  [W10] "Eta vs.          B15:  2*B13
B1:  'Zeta                   A16:  [W10] "D
C1:  '(t -                   B16:  (B14*B9*B6)+(B15*B11)
D1:  3600                    A17:  [W10] "first trm
E1:  'sec)                   B17:  +B5/@ABS(B9*B16)
F1:  "(vth^2 -               A18:  [W10] \--
G1:  2.107E-11               B18:  \-
H1:  'RU/STU)                C18:
A2:  [W10] \-                D18:
B2:                         E18:
C2:  "(Mult x                A19:  [W10] \-
D2:  0.05                    B19:  \-
E2:                         C19:  \-
F2:  '(V0 -                  D19:  \-
G2:  0.01                    E19:  \-
H2:  'm/s)                   A20:  [W10] \-
A3:  [W10] 'Constants        B20:  \-
B3:  "- -                    C20:  \-
C3:  '(W Part -              D20:  \-
D3:  1000000000000          E20:  \--
E3:  )                       A21:  [W10] 'X1(1)
F3:  'Centroid               B21:  (-7.099231E-05*D?)+G4
G3:  'X1                     A24:  [W10] "P1
H3:  "Eta                    A25:  [W10] \-
A4:  [W10] \-                B25:  \-
B4:  \-                      A26:  [W10] "P11
C4:  \-                      B26:  ((B14*B9)-(B13*B9))*B21
D4:  '- - -                  A27:  [W10] "P1(tot)
E4:  '(in RUs)               B27:  (-7.130463718E-05*$D$2)+$H$4
F4:                         C27:  (-4.753642470E-05*$D$2)+$H$4
G4:  0                       D27:  (-2.376821239E-05*$D$2)+$H$4
H4:  0                       E27:  (0*$D$2)+$H$4
A5:  [W10] "X                F27:  (2.376821239E-05*$D$2)+$H$4
B5:  1                       G27:  (4.753642470E-05*$D$2)+$H$4
A6:  [W10] "n (rad/s)        H27:  (7.130463718E-05*$D$2)+$D$2
B6:  7.315288954E-05         B28:  \-
A7:  [W10] "t                C28:  \-
B7:  +D1                     D28:  \-
A8:  [W10] "Tau              E28:  \-
B8:  +B5*B7                  F28:  \-
A9:  [W10] "s                G28:  \-
B9:  @SIN(B8)                H28:  \-
A10: [W10] "c                829:  (1/$B$16)*($B$26-($B$12*$B$11*H$27))
B10: @COS(B8)                C29:  (1/$B$16)*($B$26-($B$12*$B$11*C$27))
A11: [W10] '1-c              D29:  (1/$B$ 6)*($B$26-($B$12*$B$11*D$27))
B11: 1-B10                   E29:  (1/$B$16)*($B$26-($B$12*$B$11*E$27))
A12: [W10] "sigma            F29:  (1/$B$16)*($B$26-($B$12*$B$11*F$27))
B12: 2                       G29:  (1/$B$16)*($B$26-($B$12*$B$11*G$27))
A13: [W10] "sgmsqrd          H29:  (1/$B$16)*($B$26-($B$12*$B$11*H$27))
B13: +B12^2                  A31:  [W10] "P2
A14: [W10] "1-sgmsqrd        A32:  [W10] \-
B14: 1-B13                   B32:  \-
A15: [W10] "2*sgmsqrd        A33:  [W10] "P21
```

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```
B33:  +B12*B11*B21
B35:  +$B$27
C35:  +$C$27
D35:  +$D$27
E35:  +$E$27
F35:  +$F$27
G35:  +$G$27
H35:  +$H$27
B36:  /-
C36:  /-
D36:  /-
E36:  /-
F36:  /-
G36:  /-
H36:  /-
B37:  (1/$B$16)*($B$33+($B$9*B$35))
C37:  (1/$B$16)*($B$33+($B$9*C$35))
D37:  (1/$B$16)*($B$33+($B$9*D$35))
E37:  (1/$B$16)*($B$33+($B$9*E$35))
F37:  (1/$B$16)*($B$33+($B$9*F$35))
G37:  (1/$B$16)*($B$33+($B$9*G$35))
H37:  (1/$B$16)*($B$33+($B$9*H$35))
A39:  [W10] .P3
A40:  [W10] /-
B40:  /-
A43:  [W10] +$B$27
B43:  ($B$5/$B$9)*$A43
A44:  [W10] +$C$27
B44:  ($B$5/$B$9)*$A44
A45:  [W10] +$D$27
B45:  ($B$5/$B$9)*$A45
A46:  [W10] +$E$27
B46:  ($B$5/$B$9)*$A46
A47:  [W10] +$F$27
B47:  ($B$5/$B$9)*$A47
A48:  [W10] +$G$27
B48:  ($B$5/$B$9)*$A48
A49:  [W10] +$H$27
B49:  ($B$5/$B$9)*$A49
A51:  [W10] 'Logic table
A52:  [W10] /-
B52:  +$B$27
C53:  +$C$27
D53:  +$D$27
E53:  +$E$27
F53:  +$F$27
G53:  +$G$27
H53:  +$H$27
B54:  /-
C54:  /-
D54:  /-
E54:  /-

F54:  /-
G54:  /-
H54:  /-
A55:  [W10] +$B$27
B55:  @IF(((B$29^2+B$37^2+$B$3^2)/($G$1))>>-200,0,1)
C55:  @IF(((C$29^2+C$37^2+$B$3^2)/($G$1))>>-200,0,1)
D55:  @IF(((D$29^2+D$37^2+$B$3^2)/($G$1))>>-200,0,1)
E55:  @IF(((E$29^2+E$37^2+$B$3^2)/($G$1))>>-200,0,1)
F55:  @IF(((F$29^2+F$37^2+$B$3^2)/($G$1))>>-200,0,1)
G55:  @IF(((G$29^2+G$37^2+$B$3^2)/($G$1))>>-200,0,1)
H55:  @IF(((H$29^2+H$37^2+$B$3^2)/($G$1))>>-200,0,1)
A56:  [W10] +$C$27
B56:  @IF(((B$29^2+B$37^2+$B$4^2)/($G$1))>>-200,0,1)
C56:  @IF(((C$29^2+C$37^2+$B$4^2)/($G$1))>>-200,0,1)
D56:  @IF(((D$29^2+D$37^2+$B$4^2)/($G$1))>>-200,0,1)
E56:  @IF(((E$29^2+E$37^2+$B$4^2)/($G$1))>>-200,0,1)
F56:  @IF(((F$29^2+F$37^2+$B$4^2)/($G$1))>>-200,0,1)
G56:  @IF(((G$29^2+G$37^2+$B$4^2)/($G$1))>>-200,0,1)
H56:  @IF(((H$29^2+H$37^2+$B$4^2)/($G$1))>>-200,0,1)
A57:  [W10] +$D$27
B57:  @IF(((B$29^2+B$37^2+$B$5^2)/($G$1))>>-200,0,1)
C57:  @IF(((C$29^2+C$37^2+$B$5^2)/($G$1))>>-200,0,1)
D57:  @IF(((D$29^2+D$37^2+$B$5^2)/($G$1))>>-200,0,1)
E57:  @IF(((E$29^2+E$37^2+$B$5^2)/($G$1))>>-200,0,1)
F57:  @IF(((F$29^2+F$37^2+$B$5^2)/($G$1))>>-200,0,1)
G57:  @IF(((G$29^2+G$37^2+$B$5^2)/($G$1))>>-200,0,1)
H57:  @IF(((H$29^2+H$37^2+$B$5^2)/($G$1))>>-200,0,1)
A58:  [W10] +$E$27
B58:  @IF(((B$29^2+B$37^2+$B$6^2)/($G$1))>>-200,0,1)
C58:  @IF(((C$29^2+C$37^2+$B$6^2)/($G$1))>>-200,0,1)
D58:  @IF(((D$29^2+D$37^2+$B$6^2)/($G$1))>>-200,0,1)
E58:  @IF(((E$29^2+E$37^2+$B$6^2)/($G$1))>>-200,0,1)
F58:  @IF(((F$29^2+F$37^2+$B$6^2)/($G$1))>>-200,0,1)
G58:  @IF(((G$29^2+G$37^2+$B$6^2)/($G$1))>>-200,0,1)
H58:  @IF(((H$29^2+H$37^2+$B$6^2)/($G$1))>>-200,0,1)
A59:  [W10] +$F$27
B59:  @IF(((B$29^2+B$37^2+$B$7^2)/($G$1))>>-200,0,1)
C59:  @IF(((C$29^2+C$37^2+$B$7^2)/($G$1))>>-200,0,1)
D59:  @IF(((D$29^2+D$37^2+$B$7^2)/($G$1))>>-200,0,1)
E59:  @IF(((E$29^2+E$37^2+$B$7^2)/($G$1))>>-200,0,1)
F59:  @IF(((F$29^2+F$37^2+$B$7^2)/($G$1))>>-200,0,1)
G59:  @IF(((G$29^2+G$37^2+$B$7^2)/($G$1))>>-200,0,1)
H59:  @IF(((H$29^2+H$37^2+$B$7^2)/($G$1))>>-200,0,1)
A60:  [W10] +$G$27
B60:  @IF(((B$29^2+B$37^2+$B$8^2)/($G$1))>>-200,0,1)
C60:  @IF(((C$29^2+C$37^2+$B$8^2)/($G$1))>>-200,0,1)
D60:  @IF(((D$29^2+D$37^2+$B$8^2)/($G$1))>>-200,0,1)
E60:  @IF(((E$29^2+E$37^2+$B$8^2)/($G$1))>>-200,0,1)
F60:  @IF(((F$29^2+F$37^2+$B$8^2)/($G$1))>>-200,0,1)
G60:  @IF(((G$29^2+G$37^2+$B$8^2)/($G$1))>>-200,0,1)
H60:  @IF(((H$29^2+H$37^2+$B$8^2)/($G$1))>>-200,0,1)
A61:  [W10] +$H$27
B61:  @IF(((B$29^2+B$37^2+$B$9^2)/($G$1))>>-200,0,1)
```

Left column:

```
C61:  @IF(((CS29^2+CS37^2+$B49^2)/($G$1))>>-200,0,1)
D61:  @IF(((DS29^2+DS37^2+$B49^2)/($G$1))>>-200,0,1)
E61:  @IF(((ES29^2+ES37^2+$B49^2)/($G$1))>>-200,0,1)
F61:  @IF(((FS29^2+FS37^2+$B49^2)/($G$1))>>-200,0,1)
G61:  @IF(((GS29^2+GS37^2+$B49^2)/($G$1))>>-200,0,1)
H61:  @IF(((HS29^2+HS37^2+$B49^2)/($G$1))>>-200,0,1)
A64:  [W10]  'Rho
A65:  [W10]  \-
B65:  \-
B66:  +$B$27
C66:  +$C$27
D66:  +$D$27
E66:  +$E$27
F66:  +$F$27
G66:  +$G$27
H66:  +$H$27
B67:  \-
C67:  \-
D67:  \-
E67:  \-
F67:  \-
G67:  \-
H67:  \-
A68:  [W10]  +$B$27
B68:  @IF(B55-1,$B$17°@EXP(-(BS29^2+BS43^2)/($G$1))),0)
C68:  @IF(C55-1,$B$17°@EXP(-(CS29^2+BS43^2)/($G$1))),0)
D68:  @IF(D55-1,$B$17°@EXP(-(DS29^2+BS43^2)/($G$1))),0)
E68:  @IF(E55-1,$B$17°@EXP(-(ES29^2+BS43^2)/($G$1))),0)
F68:  @IF(F55-1,$B$17°@EXP(-(FS29^2+BS43^2)/($G$1))),0)
G68:  @IF(G55-1,$B$17°@EXP(-(GS29^2+BS43^2)/($G$1))),0)
H68:  @IF(H55-1,$B$17°@EXP(-(HS29^2+BS43^2)/($G$1))),0)
A69:  [W10]  +$C$27
B69:  @IF(B56-1,$B$17°@EXP(-(BS29^2+BS44^2)/($G$1))),0)
C69:  @IF(C56-1,$B$17°@EXP(-(CS29^2+BS44^2)/($G$1))),0)
D69:  @IF(D56-1,$B$17°@EXP(-(DS29^2+BS44^2)/($G$1))),0)
E69:  @IF(E56-1,$B$17°@EXP(-(ES29^2+BS44^2)/($G$1))),0)
F69:  @IF(F56-1,$B$17°@EXP(-(FS29^2+BS44^2)/($G$1))),0)
G69:  @IF(G56-1,$B$17°@EXP(-(GS29^2+BS44^2)/($G$1))),0)
H69:  @IF(H56-1,$B$17°@EXP(-(HS29^2+BS44^2)/($G$1))),0)
A70:  [W10]  +$D$27
B70:  @IF(B57-1,$B$17°@EXP(-(BS29^2+BS45^2)/($G$1))),0)
C70:  @IF(C57-1,$B$17°@EXP(-(CS29^2+BS45^2)/($G$1))),0)
D70:  @IF(D57-1,$B$17°@EXP(-(DS29^2+BS45^2)/($G$1))),0)
E70:  @IF(E57-1,$B$17°@EXP(-(ES29^2+BS45^2)/($G$1))),0)
F70:  @IF(F57-1,$B$17°@EXP(-(FS29^2+BS45^2)/($G$1))),0)
G70:  @IF(G57-1,$B$17°@EXP(-(GS29^2+BS45^2)/($G$1))),0)
H70:  @IF(H57-1,$B$17°@EXP(-(HS29^2+BS45^2)/($G$1))),0)
A71:  [W10]  +$E$27
B71:  @IF(B58-1,$B$17°@EXP(-(BS29^2+BS46^2)/($G$1))),0)
C71:  @IF(C58-1,$B$17°@EXP(-(CS29^2+BS46^2)/($G$1))),0)
D71:  @IF(D58-1,$B$17°@EXP(-(DS29^2+BS46^2)/($G$1))),0)
E71:  @IF(E58-1,$B$17°@EXP(-(ES29^2+BS46^2)/($G$1))),0)
F71:  @IF(F58-1,$B$17°@EXP(-(FS29^2+BS46^2)/($G$1))),0)
```

Right column:

```
G71:  @IF(G58-1,$B$17°@EXP(-(GS29^2+BS37^2+$B46^2)/($G$1))),0)
H71:  @IF(H58-1,$B$17°@EXP(-(HS29^2+BS37^2+$B46^2)/($G$1))),0)
A72:  [W10]  +$F$27
B72:  @IF(B59-1,$B$17°@EXP(-(BS29^2+BS37^2+$B47^2)/($G$1))),0)
C72:  @IF(C59-1,$B$17°@EXP(-(CS29^2+BS37^2+$B47^2)/($G$1))),0)
D72:  @IF(D59-1,$B$17°@EXP(-(DS29^2+BS37^2+$B47^2)/($G$1))),0)
E72:  @IF(E59-1,$B$17°@EXP(-(ES29^2+BS37^2+$B47^2)/($G$1))),0)
F72:  @IF(F59-1,$B$17°@EXP(-(FS29^2+BS37^2+$B47^2)/($G$1))),0)
G72:  @IF(G59-1,$B$17°@EXP(-(GS29^2+BS37^2+$B47^2)/($G$1))),0)
H72:  @IF(H59-1,$B$17°@EXP(-(HS29^2+BS37^2+$B47^2)/($G$1))),0)
A73:  [W10]  +$G$27
B73:  @IF(B60-1,$B$17°@EXP(-(BS29^2+BS37^2+$B48^2)/($G$1))),0)
C73:  @IF(C60-1,$B$17°@EXP(-(CS29^2+BS37^2+$B48^2)/($G$1))),0)
D73:  @IF(D60-1,$B$17°@EXP(-(DS29^2+DS37^2+$B48^2)/($G$1))),0)
E73:  @IF(E60-1,$B$17°@EXP(-(ES29^2+ES37^2+$B48^2)/($G$1))),0)
F73:  @IF(F60-1,$B$17°@EXP(-(FS29^2+FS37^2+$B48^2)/($G$1))),0)
G73:  @IF(G60-1,$B$17°@EXP(-(GS29^2+GS37^2+$B48^2)/($G$1))),0)
H73:  @IF(H60-1,$B$17°@EXP(-(HS29^2+HS37^2+$B48^2)/($G$1))),0)
A74:  [W10]  +$H$27
B74:  @IF(B61-1,$B$17°@EXP(-(BS29^2+BS37^2+$B49^2)/($G$1))),0)
C74:  @IF(C61-1,$B$17°@EXP(-(CS29^2+CS37^2+$B49^2)/($G$1))),0)
D74:  @IF(D61-1,$B$17°@EXP(-(DS29^2+DS37^2+$B49^2)/($G$1))),0)
E74:  @IF(E61-1,$B$17°@EXP(-(ES29^2+ES37^2+$B49^2)/($G$1))),0)
F74:  @IF(F61-1,$B$17°@EXP(-(FS29^2+FS37^2+$B49^2)/($G$1))),0)
G74:  @IF(G61-1,$B$17°@EXP(-(GS29^2+GS37^2+$B49^2)/($G$1))),0)
H74:  @IF(H61-1,$B$17°@EXP(-(HS29^2+HS37^2+$B49^2)/($G$1))),0)
A75:  [W10]  \-
B75:  \-
C75:  \-
D75:  \-
E75:  \-
F75:  \-
G75:  \-
H75:  \-
A77:  [W10]  \--
B77:  \--
A78:  [W10]  \--
B78:  \--
A80:  [W10]  'X1(C2)
B80:  (-4.732815%E-05-D2)+G4
A83:  [W10]  'P1
A84:  [W10]  \--
B84:  \--
A85:  [W10]  "P11
B85:  ((B14*B8)+(B13*B9))+B80
A86:  [W10]  "P1(tot)
B86:  +$B$27
C86:  +$C$27
D86:  +$D$27
E86:  +$E$27
F86:  +$F$27
G86:  +$G$27
H86:  +$H$27
```

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```
B87.     /-
C87:     /-
D87.     /-
E67:     /-
F87:     /-
G87:     /-
H87:     /-
B88:     (1/$B$16)*($B$85-($B$12*$B$11*B$86))
C88:     (1/$B$16)*($B$85-($B$12*$B$11*C$86))
D88:     (1/$B$16)*($B$85-($B$12*$B$11*D$86))
E88:     (1/$B$16)*($B$85-($B$12*$B$11*E$86))
F88:     (1/$B$16)*($B$85-($B$12*$B$11*F$86))
G88:     (1/$B$16)*($B$85-($B$12*$B$11*G$86))
H88:     (1/$B$16)*($B$85-($B$12*$B$11*H$86))
A90:     [W10] "P2
A91:     [W10] /-
B91:     /-
A92:     [W10] "P21
B92:     +B12*B11*B80
B94:     +$B$27
C94:     +$C$27
D94:     +$D$27
E94:     +$E$27
F94:     +$F$27
G94:     +$G$27
H94:     +$H$27
B95:     /-
C95:     /-
D95:     /-
E95:     /-
F95:     /-
G95:     /-
H95:     /-
B96:     (1/$B$16)*($B$92+($B$9*B$94))
C96:     (1/$B$16)*($B$92+($B$9*C$94))
D96:     (1/$B$16)*($B$92+($B$9*D$94))
E96:     (1/$B$16)*($B$92+($B$9*E$94))
F96:     (1/$B$16)*($B$92+($B$9*F$94))
G96:     (1/$B$16)*($B$92+($B$9*G$94))
H96:     (1/$B$16)*($B$92+($B$9*H$94))
A98:     [W10] "P3
A99:     [W10] /-
B99:     /-
A102:    [W10] +$B$27
B102:    ($B$5/$B$9)*$A102
A103:    [W10] +$C$27
B103:    ($B$5/$B$9)*$A103
A104:    [W10] +$D$27
B104:    ($B$5/$B$9)*$A104
A105:    [W10] +$E$27
B105:    ($B$5/$B$9)*$A105
A106:    [W10] +$F$27
B106:    ($B$5/$B$9)*$A106

A107:    [W10] +$G$27
B107:    ($B$5/$B$9)*$A107
A108:    [W10] +$H$27
B108:    ($B$5/$B$9)*$A108
A110:    [W10] 'Logic table
A111:    [W10] /-
B111:    /-
B112:    +$B$27
C112:    +$C$27
D112:    +$D$27
E112:    +$E$27
F112:    +$F$27
G112:    +$G$27
H112:    +$H$27
B113:    /-
C113:    /-
D113:    /-
F113:    /-
G113:    /-
H113:    /-
A114:    [W10] +$B$27
B114:    @IF((($B$88^2+B$96^2+$B$102^2)/($G$1))>>=200,0,1)
C114:    @IF((($C$88^2+C$96^2+$D$102^2)/($G$1))>>=200,0,1)
D114:    @IF((($D$88^2+D$96^2+$B$102^2)/($G$1))>>=200,0,1)
E114:    @IF((($E$88^2+E$96^2+$B$102^2)/($G$1))>>=200,0,1)
F114:    @IF((($F$88^2+F$96^2+$B$102^2)/($G$1))>>=200,0,1)
G114:    @IF((($G$88^2+G$96^2+$B$102^2)/($G$1))>>=200,0,1)
H114:    @IF((($H$88^2+H$96^2+$B$102^2)/($G$1))>>=200,0,1)
A115:    [W10] +$C$27
B115:    @IF((($B$88^2+B$96^2+$B$103^2)/($G$1))>>=200,0,1)
C115:    @IF((($C$88^2+C$96^2+$B$103^2)/($G$1))>>=200,0,1)
D115:    @IF((($D$88^2+D$96^2+$C$103^2)/($G$1))>>=200,0,1)
E115:    @IF((($E$88^2+E$96^2+$B$103^2)/($G$1))>>=200,0,1)
F115:    @IF((($F$88^2+F$96^2+$B$103^2)/($G$1))>>=200,0,1)
G115:    @IF((($G$88^2+G$96^2+$B$103^2)/($G$1))>>=200,0,1)
H115:    @IF((($H$88^2+H$96^2+$B$103^2)/($G$1))>>=200,0,1)
A116:    [W10] +$D$27
B116:    @IF((($B$88^2+B$96^2+$B$104^2)/($G$1))>>=200,0,1)
C116:    @IF((($C$88^2+C$96^2+$B$104^2)/($G$1))>>=200,0,1)
D116:    @IF((($D$88^2+D$96^2+$B$104^2)/($G$1))>>=200,0,1)
E116:    @IF((($E$88^2+E$96^2+$B$104^2)/($G$1))>>=200,0,1)
F116:    @IF((($F$88^2+F$96^2+$B$104^2)/($G$1))>>=200,0,1)
G116:    @IF((($G$88^2+G$96^2+$B$104^2)/($G$1))>>=200,0,1)
H116:    @IF((($H$88^2+H$96^2+$B$104^2)/($G$1))>>=200,0,1)
A117:    [W10] +$E$27
B117:    @IF((($B$89^2+B$96^2+$B$105^2)/($G$1))>>=200,0,1)
C117:    @IF((($C$88^2+C$96^2+$B$105^2)/($G$1))>>=200,0,?)
D117:    @IF((($D$88^2+D$96^2+$B$105^2)/($G$1))>>=200,0,1)
E117:    @IF((($E$88^2+E$96^2+$B$105^2)/($G$1))>>=200,0,1)
F117:    @IF((($F$88^2+F$96^2+$B$105^2)/($G$1))>>=200,0,1)
G117:    @IF((($G$88^2+G$96^2+$B$105^2)/($G$1))>>=200,0,1)
H117:    @IF((($H$88^2+H$96^2+$B$105^2)/($G$1))>>=200,0,1)
```

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```
A118: [W10] +$F$27
B118:
C118: @IF(((C$88^2+B$96^2+$B106^2)/($G$1))>-200,0,1)
D118: @IF(((D$88^2+C$96^2+$B106^2)/($G$1))>-200,0,1)
E116: @IF(((E$88^2+D$96^2+$B106^2)/($G$1))>-200,0,1)
F118: @IF(((F$88^2+E$96^2+$B106^2)/($G$1))>-200,0,1)
G118: @IF(((G$88^2+F$96^2+$B106^2)/($G$1))>-200,0,1)
H118: @IF(((H$88^2+G$96^2+$B106^2)/($G$1))>-200,0,1)
A119: [W10] +$G$27
B119: @IF(((B$88^2+B$96^2+$B107^2)/($G$1))>-200,0,1)
C119: @IF(((C$88^2+C$96^2+$B107^2)/($G$1))>-200,0,1)
D119: @IF(((D$88^2+D$96^2+$B107^2)/($G$1))>-200,0,1)
E119: @IF(((E$88^2+E$96^2+$B107^2)/($G$1))>-200,0,1)
F119: @IF(((F$88^2+F$96^2+$B107^2)/($G$1))>-200,0,1)
G119: @IF(((G$88^2+G$96^2+$B107^2)/($G$1))>-200,0,1)
H119: @IF(((H$88^2+H$96^2+$B107^2)/($G$1))>-200,0,1)
A120: [W10] +$H$27
B120: @IF(((B$88^2+B$96^2+$B108^2)/($G$1))>-200,0,1)
C120: @IF(((C$88^2+C$96^2+$B108^2)/($G$1))>-200,0,1)
D120: @IF(((D$88^2+D$96^2+$B108^2)/($G$1))>-200,0,1)
E120: @IF(((E$88^2+E$96^2+$B108^2)/($G$1))>-200,0,1)
F120: @IF(((F$88^2+F$96^2+$B108^2)/($G$1))>-200,0,1)
G120: @IF(((G$88^2+G$96^2+$B108^2)/($G$1))>-200,0,1)
H120: @IF(((H$88^2+H$96^2+$B108^2)/($G$1))>-200,0,1)
A123: [W10] 'Rho
A124: [W10] \-
B124: \-
B125: +$B$27
C125: +$C$27
D125: +$D$27
E125: +$E$27
F125: +$F$27
G125: +$G$27
H125: +$H$27
B126: \-
C126: \-
D126: \-
E126: \-
F126: \-
G126: \-
H126: \-
A127: [W10] +$B$27
B127:
@IF(B114=1,$B$17*@EXP(-(G$88^2+B$96^2+$B102^2)/($G$1))),0)
C127:
@IF(C114=1,$B$17*@EXP(-(C$88^2+C$96^2+$B102^2)/($G$1))),0)
D127:
@IF(D114=1,$B$17*@EXP(-(D$88^2+D$96^2+$B102^2)/($G$1))),0)
E127:
@IF(E114=1,$B$17*@EXP(-(E$88^2+E$96^2+$B102^2)/($G$1))),0)
F127:
@IF(F114=1,$B$17*@EXP(-(F$88^2+F$96^2+$B102^2)/($G$1))),0)
G127:

@IF(G114=1,$B$17*@EXP(-(G$88^2+G$96^2+$B102^2)/($G$1))),0)
H127:
A128: [W10] +$C$27
B128:
@IF(B115=1,$B$17*@EXP(-(B$88^2+B$96^2+$B103^2)/($G$1))),0)
C128:
@IF(C115=1,$B$17*@EXP(-(C$88^2+C$96^2+$B103^2)/($G$1))),0)
D128:
@IF(D115=1,$B$17*@EXP(-(D$88^2+D$96^2+$B103^2)/($G$1))),0)
E128:
@IF(E115=1,$B$17*@EXP(-(E$88^2+E$96^2+$B103^2)/($G$1))),0)
F128:
@IF(F115=1,$B$17*@EXP(-(F$88^2+F$96^2+$B103^2)/($G$1))),0)
G128:
@IF(G115=1,$B$17*@EXP(-(G$88^2+G$96^2+$B103^2)/($G$1))),0)
H128:
@IF(H115=1,$B$17*@EXP(-(H$88^2+H$96^2+$B103^2)/($G$1))),0)
A129: [W10] +$D$27
B129:
C129:
@IF(B116=1,$B$17*@EXP(-(B$88^2+B$96^2+$B104^2)/($G$1))),0)
C129:
@IF(C116=1,$B$17*@EXP(-(C$88^2+C$96^2+$B104^2)/($G$1))),0)
D129:
@IF(D116=1,$B$17*@EXP(-(D$88^2+D$96^2+$B104^2)/($G$1))),0)
E129:
@IF(E116=1,$B$17*@EXP(-(E$88^2+E$96^2+$B104^2)/($G$1))),0)
F129:
@IF(F116=1,$B$17*@EXP(-(F$88^2+F$96^2+$B104^2)/($G$1))),0)
G129:
@IF(G116=1,$B$17*@EXP(-(G$88^2+G$96^2+$B104^2)/($G$1))),0)
H129:
@IF(H116=1,$B$17*@EXP(-(H$88^2+H$96^2+$B104^2)/($G$1))),0)
A130: [W10] +$E$27
B130:
@IF(B117=1,$B$17*@EXP(-(B$88^2+B$96^2+$B105^2)/($G$1))),0)
C130:
@IF(C117=1,$B$17*@EXP(-(C$88^2+C$96^2+$B105^2)/($G$1))),0)
D130:
@IF(D117=1,$B$17*@EXP(-(D$88^2+D$96^2+$B105^2)/($G$1))),0)
E130:
@IF(E117=1,$B$17*@EXP(-(E$88^2+E$96^2+$B105^2)/($G$1))),0)
F130:
@IF(F117=1,$B$17*@EXP(-(F$88^2+F$96^2+$B105^2)/($G$1))),0)
G130:
@IF(G117=1,$B$17*@EXP(-(G$88^2+G$96^2+$B105^2)/($G$1))),0)
H130:
@IF(H117=1,$B$17*@EXP(-(H$88^2+H$96^2+$B105^2)/($G$1))),0)
A131: [W10] +$F$27
B131:
@IF(B118=1,$B$17*@EXP(-(B$88^2+B$96^2+$B106^2)/($G$1))),0)
C131:
@IF(C118=1,$B$17*@EXP(-(C$88^2+C$96^2+$B106^2)/($G$1))),0)
```

```
D131:
@IF(D118=1,$B$17*@EXP(-(D$88^2+D$96^2+$B106^2)/($G$1)),0)
E134:
@IF(E118=1,$B$17*@EXP(-(E$88^2+E$96^2+$B106^2)/($G$1)),0)
F131:
@IF(F118=1,$B$17*@EXP(-(F$88^2+F$96^2+$B106^2)/($G$1)),0)
G131:
@IF(G118=1,$B$17*@EXP(-(G$88^2+G$96^2+$B106^2)/($G$1)),0)
H131:
@IF(H118=1,$B$17*@EXP(-(H$88^2+H$96^2+$B106^2)/($G$1)),0)
A132: [W10] +$G$27
B132:
@IF(B119=1,$B$17*@EXP(-(B$88^2+B$96^2+$B107^2)/($G$1)),0)
C132:
@IF(C119=1,$B$17*@EXP(-(C$88^2+C$96^2+$B107^2)/($G$1)),0)
D132:
@IF(D119=1,$B$17*@EXP(-(D$88^2+D$96^2+$B107^2)/($G$1)),0)
E132:
@IF(E119=1,$B$17*@EXP(-(E$88^2+E$96^2+$B107^2)/($G$1)),0)
F132:
@IF(F119=1,$B$17*@EXP(-(F$88^2+F$96^2+$B107^2)/($G$1)),0)
G132:
@IF(G119=1,$B$17*@EXP(-(G$88^2+G$96^2+$B107^2)/($G$1)),0)
H132:
@IF(H119=1,$B$17*@EXP(-(H$88^2+H$96^2+$B107^2)/($G$1)),0)
A133: [W10] +$H$27
B133:
@IF(B120=1,$B$17*@EXP(-(B$88^2+B$96^2+$B108^2)/($G$1)),0)
C133:
@IF(C120=1,$B$17*@EXP(-(C$88^2+C$96^2+$B108^2)/($G$1)),0)
D133:
@IF(D120=1,$B$17*@EXP(-(D$88^2+D$96^2+$B108^2)/($G$1)),0)
E133:
@IF(E120=1,$B$17*@EXP(-(E$88^2+E$96^2+$B108^2)/($G$1)),0)
F133:
@IF(F120=1,$B$17*@EXP(-(F$88^2+F$96^2+$B108^2)/($G$1)),0)
G133:
@IF(G120=1,$B$17*@EXP(-(G$88^2+G$96^2+$B108^2)/($G$1)),0)
X133:
@IF(H120=1,$B$17*@EXP(-(H$88^2+H$96^2+$B108^2)/($G$1)),0)
A134: [W10] \-
B134: \-
C134: \-
D134: \-
E134: \-
F134: \-
G134:
H134: [W10] \-
A136: [W10] \--
B136: \-
A137: [W10] \-
B137: \-
A139: [W10] `X1(3)

B139: (-2.3664077E-05*D2)+G4
A142: [W10] `P1
A143: [W10] \-
B143: \-
A144: [W10] `P11
B144: ((B14*B8)+(B13*B9))*B139
A145: [W10] `P1(tot)
B145: +$B$27
C145: +$C$27
D145: +$D$27
E145: +$E$27
F145: +$F$27
G145: +$G$27
H145: +$H$27
B146: \-
C146: \-
D146: \-
E146: \-
F146: \-
G146: \-
H146: \-
B147: (1/$B$16)*($B$144-($B$12*$B$11*B$145))
C147: (1/$B$16)*($B$144-($B$12*$B$11*C$145))
D147: (1/$B$16)*($B$144-($B$12*$B$11*D$145))
E147: (1/$B$16)*($B$144-($B$12*$B$11*E$145))
F147: (1/$B$16)*($B$144-($B$12*$B$11*F$145))
G147: (1/$B$16)*($B$144-($B$12*$B$11*G$145))
H147: (1/$B$16)*($B$144-($B$12*$B$11*H$145))
A149: [W10] `P2
A150: [W10] \-
B150: \-
A151: [W10] `P21
B151: +B12*B11*B139
B153: +$B$27
C153: +$C$27
D153: +$D$27
E153: +$E$27
F153: +$F$27
G153: +$G$27
H153: +$H$27
B154: \-
C154: \-
E154: \-
F154: \-
N154: \-
B155: (1/$B$16)*($B$151+($B$9*B$153))
C155: (1/$B$16)*($B$151+($B$9*C$153))
D155: (1/$B$16)*($B$151+($B$9*D$153))
E155: (1/$B$16)*($B$151+($B$9*E$153))
F155: (1/$B$16)*($B$151+($B$9*F$153))
G155: (1/$B$16)*($B$151+($B$9*G$153))
```

```
H155: (1/SBS16)/(SBS151+(SBS9*HS153))
A157: [W10] 'P3
A158: [W10] \-
B158:
A161: [W10] +SBS27
B161: (SBS5/SBS9)*SA161
A162: [W10] +SCS27
B162: (SBS5/SBS9)*SA162
A163: [W10] +SDS27
B163: (SBS5/SBS9)*SA163
A164: [W10] +SES27
B164: (SBS5/SBS9)*SA164
A165: [W10] +SFS27
B165: (SBS5/SBS9)*SA165
A166: [W10] +SGS27
B166: (SBS5/SBS9)*SA166
A167: [W10] +SHS27
B167: (SBS5/SBS9)*SA167
A169: [W10] 'Logic table
A170: [W10] \-
B170: \-
B171: +SBS27
C171: +SCS27
D171: +SDS27
E171: +SES27
F171: +SFS27
G171: +SGS27
H171: +SHS27
B172: \-
C172: \-
D172: \-
E172: \-
F172: \-
G172: \-
H172: \-
A173: [W10] +SBS27
B173: @IF(((BS147^2+BS155^2+SB161^2)/(SGS1))>>=200,0,1)
C173: @IF(((CS147^2+CS155^2+SB161^2)/(SGS1))>>=200,0,1)
D173: @IF(((DS147^2+DS155^2+SB161^2)/(SGS1))>>=200,0,1)
E173: @IF(((ES147^2+ES155^2+SB161^2)/(SGS1))>>=200,0,1)
F173: @IF(((FS147^2+FS155^2+SB161^2)/(SGS1))>>=200,0,1)
G173: @IF(((GS147^2+GS155^2+SB161^2)/(SGS1))>>=200,0,1)
H173: @IF(((HS147^2+HS155^2+SB161^2)/(SGS1))>>=200,0,1)
A174: [W10] +SCS27
B174: @IF(((BS147^2+BS155^2+SB162^2)/(SGS1))>>=200,0,1)
C174: @IF(((CS147^2+CS155^2+SB162^2)/(SGS1))>>=200,0,1)
D174: @IF(((DS147^2+DS155^2+SB162^2)/(SGS1))>>=200,0,1)
E174: @IF(((ES147^2+ES155^2+SB162^2)/(SGS1))>>=200,0,1)
F174: @IF(((FS147^2+FS155^2+SB162^2)/(SGS1))>>=200,0,1)
G174: @IF(((GS147^2+GS155^2+SB162^2)/(SGS1))>>=200,0,1)
H174: @IF(((HS147^2+HS155^2+SB162^2)/(SGS1))>>=200,0,1)
A175: [W10] +SDS27
B175: @IF(((BS147^2+BS155^2+SB163^2)/(SGS1))>>=200,0,1)

C175: @IF(((CS147^2+CS155^2+SB163^2)/(SGS1))>>=200,0,1)
D175: @IF(((DS147^2+DS155^2+SB163^2)/(SGS1))>>=200,0,1)
E175: @IF(((ES147^2+ES155^2+SB163^2)/(SGS1))>>=200,0,1)
F175: @IF(((FS147^2+GS155^2+SB163^2)/(SGS1))>>=200,0,1)
G175: @IF(((GS147^2+GS155^2+SB163^2)/(SGS1))>>=200,0,1)
H175: @IF(((HS147^2+HS155^2+SB163^2)/(SGS1))>>=200,0,1)
A176: [W10] +SES27
B176: @IF(((BS147^2+BS155^2+SB164^2)/(SGS1))>>=200,0,1)
C176: @IF(((CS147^2+CS155^2+SB164^2)/(SGS1))>>=200,0,1)
D176: @IF(((DS147^2+DS155^2+SB164^2)/(SGS1))>>=200,0,1)
E176: @IF(((ES147^2+ES155^2+SB164^2)/(SGS1))>>=200,0,1)
F176: @IF(((FS147^2+FS155^2+SB164^2)/(SGS1))>>=200,0,1)
G176: @IF(((GS147^2+GS155^2+SB164^2)/(SGS1))>>=200,0,1)
H176: @IF(((HS147^2+HS155^2+SB164^2)/(SGS1))>>=200,0,1)
A177: [W10] +SFS27
B177: @IF(((BS147^2+BS155^2+SB165^2)/(SGS1))>>=200,0,1)
C177: @IF(((CS147^2+CS155^2+SB165^2)/(SGS1))>>=200,0,1)
D177: @IF(((DS147^2+DS155^2+SB165^2)/(SGS1))>>=200,0,1)
E177: @IF(((ES147^2+ES155^2+SB165^2)/(SGS1))>>=200,0,1)
F177: @IF(((FS147^2+FS155^2+SB165^2)/(SGS1))>>=200,0,1)
G177: @IF(((GS147^2+GS155^2+SB165^2)/(SGS1))>>=200,0,1)
H177: @IF(((HS147^2+HS155^2+SB165^2)/(SGS1))>>=200,0,1)
A178: [W10] +SGS27
B178: @IF(((BS147^2+BS155^2+SB166^2)/(SGS1))>>=200,0,1)
C178: @IF(((CS147^2+CS155^2+SB166^2)/(SGS1))>>=200,0,1)
D178: @IF(((DS147^2+DS155^2+SB166^2)/(SGS1))>>=200,0,1)
E178: @IF(((ES147^2+ES155^2+SB166^2)/(SGS1))>>=200,0,1)
F178: @IF(((FS147^2+FS155^2+SB166^2)/(SGS1))>>=200,0,1)
G178: @IF(((GS147^2+GS155^2+SB166^2)/(SGS1))>>=200,0,1)
H178: @IF(((HS147^2+HS155^2+SB166^2)/(SGS1))>>=200,0,1)
A173: [W10] +SHS27
B179: @IF(((BS147^2+ES155^2+SB167^2)/(SGS1))>>=200,0,1)
C179: @IF(((CS147^2+CS155^2+SB167^2)/(SGS1))>>=200,0,1)
D179: @IF(((DS147^2+DS155^2+SB167^2)/(SGS1))>>=200,0,1)
E179: @IF(((ES147^2+FS155^2+SB167^2)/(SGS1))>>=200,0,1)
F179: @IF(((FS147^2+GS155^2+SB167^2)/(SGS1))>>=200,0,1)
G179: @IF(((GS147^2+HS155^2+SB167^2)/(SGS1))>>=200,0,1)
H179: @IF(((HS147^2+HS155^2+SB167^2)/(SGS1))>>=200,0,1)
A182: [W10] 'Rho
A183: [W10] \-
B183: \-
B184: +SBS27
C184: +SCS27
D184: +SDS27
E184: +SES27
F184: +SFS27
G184: +SGS27
H184: +SHS27
B185: \-
C185: \-
D185: \-
E185: \-
F185: \-
```

G185: \-
M185: \-
A186: [W10) +$B$27
B186:
C186:
@IF(B173=1,$B$17*@EXP(-(B147^2+B155^2+$B161^2)/($G$1))),0)
D186:
@IF(C173=1,$B$17*@EXP(-(C147^2+C155^2+$B161^2)/($G$1))),0)
E186:
@IF(D173=1,$B$17*@EXP(-(D147^2+D155^2+$B161^2)/($G$1))),0)
F186:
@IF(E173=1,$B$17*@EXP(-(E147^2+E155^2+$B161^2)/($G$1))),0)
@IF(F173=1,$B$17*@EXP(-(F147^2+F155^2+$B161^2)/($G$1))),0)
G186:
@IF(G173=1,$B$17*@EXP(-(G147^2+G155^2+$B161^2)/($G$1))),0)
M186:
@IF(H173=1,$B$17*@EXP(-(H147^2+H155^2+$B161^2)/($G$1))),0)
A187: [W10) +$C$27
B187:
@IF(B174=1,$B$17*@EXP(-(B147^2+B155^2+$B162^2)/($G$1))),0)
C187:
@IF(C174=1,$B$17*@EXP(-(C147^2+C155^2+$B162^2)/($G$1))),0)
D187:
@IF(D174=1,$B$17*@EXP(-(D147^2+D155^2+$B162^2)/($G$1))),0)
E187:
@IF(E174=1,$B$17*@EXP(-(E147^2+E155^2+$B162^2)/($G$1))),0)
F187:
@IF(F174=1,$B$17*@EXP(-(F147^2+F155^2+$B162^2)/($G$1))),0)
G187:
@IF(G174=1,$B$17*@EXP(-(G147^2+G155^2+$B162^2)/($G$1))),0)
M187:
@IF(H174=1,$B$17*@EXP(-(H147^2+H155^2+$B162^2)/($G$1))),0)
A188: [W10) +$D$27
B188:
@IF(B175=1,$B$17*@EXP(-(B147^2+B155^2+$B163^2)/($G$1))),0)
C188:
@IF(C175=1,$B$17*@EXP(-(C147^2+C155^2+$B163^2)/($G$1))),0)
D188:
@IF(D175=1,$B$17*@EXP(-(D147^2+D155^2+$B163^2)/($G$1))),0)
E188:
@IF(E175=1,$B$17*@EXP(-(E147^2+E155^2+$B163^2)/($G$1))),0)
F188:
@IF(F175=1,$B$17*@EXP(-(F147^2+F155^2+$B163^2)/($G$1))),0)
G188:
@IF(G175=1,$B$17*@EXP(-(G147^2+G155^2+$B163^2)/($G$1))),0)
M188:
@IF(H175=1,$B$17*@EXP(-(H147^2+H155^2+$B163^2)/($G$1))),0)
A189: [W10) +$E$27
B189:
@IF(B176=1,$B$17*@EXP(-(B147^2+B155^2+$B164^2)/($G$1))),0)
C189:
@IF(C176=1,$B$17*@EXP(-(C147^2+C155^2+$B164^2)/($G$1))),0)
D189:

@IF(D176=1,$B$17*@EXP(-(D147^2+D155^2+$B164^2)/($G$1))),0)
E189:
@IF(E176=1,$B$17*@EXP(-(E147^2+E155^2+$B164^2)/($G$1))),0)
F189:
@IF(F176=1,$B$17*@EXP(-(F147^2+F155^2+$B164^2)/($G$1))),0)
G189:
@IF(G176=1,$B$17*@EXP(-(G147^2+G155^2+$B164^2)/($G$1))),0)
M189
@IF(H176=1,$B$17*@EXP(-(H147^2+H155^2+$B164^2)/($G$1))),0)
A190: [W10) +$F$27
B190:
@IF(B177=1,$B$17*@EXP(-(B147^2+B155^2+$B165^2)/($G$1))),0)
C190:
@IF(C177=1,$B$17*@EXP(-(C147^2+C155^2+$B165^2)/($G$1))),0)
D190:
@IF(D177=1,$B$17*@EXP(-(D147^2+D155^2+$B165^2)/($G$1))),0)
E190:
@IF(E177=1,$B$17*@EXP(-(E147^2+E155^2+$B165^2)/($G$1))),0)
F190:
@IF(F177=1,$B$17*@EXP(-(F147^2+F155^2+$B165^2)/($G$1))),0)
G190:
@IF(G177=1,$B$17*@EXP(-(G147^2+G155^2+$B165^2)/($G$1))),0)
M190:
@IF(H177=1,$B$17*@EXP(-(H147^2+H155^2+$B165^2)/($G$1))),0)
A191: [W10) +$G$27
B191:
@IF(B178=1,$B$17*@EXP(-(B147^2+B155^2+$B166^2)/($G$1))),0)
C191:
@IF(C178=1,$B$17*@EXP(-(C147^2+C155^2+$B166^2)/($G$1))),0)
D191:
@IF(D178=1,$B$17*@EXP(-(D147^2+D155^2+$B166^2)/($G$1))),0)
E191:
@IF(E178=1,$B$17*@EXP(-(E147^2+E155^2+$B166^2)/($G$1))),0)
F191:
@IF(F178=1,$B$17*@EXP(-(F147^2+F155^2+$B166^2)/($G$1))),0)
G191:
@IF(G178=1,$B$17*@EXP(-(G147^2+G155^2+$B166^2)/($G$1))),0)
M191:
@IF(H178=1,$B$17*@EXP(-(H147^2+H155^2+$B166^2)/($G$1))),0)
A192: [W10) +$H$27
B192:
@IF(B179=1,$B$17*@EXP(-(B147^2+B155^2+$B167^2)/($G$1))),0)
C192:
@IF(C179=1,$B$17*@EXP(-(C147^2+C155^2+$B167^2)/($G$1))),0)
D192:
@IF(D179=1,$B$17*@EXP(-(D147^2+D155^2+$B167^2)/($G$1))),0)
E192:
@IF(E179=1,$B$17*@EXP(-(E147^2+E155^2+$B167^2)/($G$1))),0)
F192:
@IF(F179=1,$B$17*@EXP(-(F147^2+F155^2+$B167^2)/($G$1))),0)
G192:
@IF(G179=1,$B$17*@EXP(-(G147^2+G155^2+$B167^2)/($G$1))),0)
M192:

```
@IF(H179=1,$B$17*@EXP(-(H$147^2+H$155^2+$B$167^2)/($G$1)),0)
A193: [W10] \--
B193: \--
C193: \--
D193: \--
E193: \--
F193: \--
G193: \--
H193: \--
A195: [W10] \--
B195: \--
A196: [W10] \--
B196: \--
A198: [W10] "X1(4)
B198: (0*D2)+G4
A201: [W10] "P1
A202: [W10] \--
B202: \--
A203: [W10] "P11
B203: ((B14*B8)+(B13*B9))*B198
A204: [W10] "P1(tot)
B204: +$B$27
C204: +$C$27
D204: +$D$27
E204: +$E$27
F204: +$F$27
G204: +$G$27
H204: +$H$27
A205: \--
B205: \--
C205: \--
D205: \--
E205: \--
F205: \--
G205: \--
H205: \--
B206: (1/$B$16)*($B$203-($B$12*$B$11*B$204))
C206: (1/$B$16)*($B$203-($B$12*$B$11*C$204))
D206: (1/$B$16)*($B$203-($B$12*$B$11*D$204))
E206: (1/$B$16)*($B$203-($B$12*$B$11*E$204))
F206: (1/$B$16)*($B$203-($B$12*$B$11*F$204))
G206: (1/$B$16)*($B$203-($B$12*$C$11*G$204))
H206: (1/$B$16)*($B$203-($B$12*$B$11*H$204))
A208: [W10] "P2
A209: [W10] \--
B209: \--
A210: [W10] "P21
B210: +B12*B11*B198
B212: +$B$27
C212: +$C$27
D212: +$D$27
E212: +$E$27
F212: +$F$27
G212: +$G$27

H212: +$H$27
B213: \--
C213: \--
D213: \--
E213: \--
F213: \--
G213: \--
H213: \--
B214: (1/$B$16)*($B$210+($B$9*B$212))
C214: (1/$B$16)*($B$210+($B$9*C$212))
D214: (1/$B$16)*($B$210+($B$9*D$212))
E214: (1/$B$16)*($B$210+($B$9*E$212))
F214: (1/$B$16)*($B$210+($B$9*F$212))
G214: (1/$B$16)*($B$210+($B$9*G$212))
H214: (1/$B$16)*($B$210+($B$9*H$212))
A216: [W10] "P3
A217: [W10] \--
B217: \--
A220: [W10] +$B$27
B220: ($B$5/$B$9)*$A220
A221: [W10] +$C$27
B221: ($B$5/$B$9)*$A221
A222: [W10] +$D$27
B222: ($B$5/$B$9)*$A222
A223: [W10] +$E$27
B223: ($B$5/$B$9)*$A223
A224: [W10] +$F$27
B224: ($B$5/$B$9)*$A224
A225: [W10] +$G$27
B225: ($B$5/$B$9)*$A225
A226: [W10] +$H$27
B226: ($B$5/$B$9)*$A226
A228: [W10] 'Logic table
A229: [W10] \--
B229: \--
A230: +$B$27
C230: +$C$27
D230: +$D$27
E230: +$E$27
F230: +$F$27
G230: +$G$27
H230: +$H$27
B231: \--
C231: \--
D231: \--
E231: \--
F231: \--
G231: \--
H231: \--
A232: [W10] +$B$27
R232: @IF(((B$206^2+B$214^2+$B$220^2)/($G$1))>=200,0,1)
C232: @IF(((C$206^2+C$214^2+$B$220^2)/($G$1))>=200,0,1)
D232: @IF(((D$206^2+D$214^2+$B$220^2)/($G$1))>=200,0,1)
```

Left column:

```
E247:
@IF(C234=1,$BS17*@EXP(-(ES206^2+ES214^2+$B222^2)/($GS1)),0)
F247:
@IF(F234=1,$BS17*@EXP(-(FS206^2+FS214^2+$B222^2)/($GS1)),0)
G247:
@IF(G234=1,$BS17*@EXP(-(GS206^2+GS214^2+$B222^2)/($GS1)),0)
H247:
@IF(H234=1,$BS17*@EXP(-(HS206^2+HS214^2+$B222^2)/($GS1)),0)
A248: [W10] +SES27
B248:
@IF(B235=1,$BS17*@EXP(-(BS206^2+BS214^2+$B223^2)/($GS1)),0)
C248:
@IF(C235=1,$BS17*@EXP(-(CS206^2-CS214^2+$B223^2)/($GS1)),0)
D248:
@IF(D235=1,$BS17*@EXP(-(DS206^2+DS214^2+$B223^2)/($GS1)),0)
E248:
@IF(E235=1,$BS17*@EXP(-(ES206^2+ES214^2+$B223^2)/($GS1)),0)
F248:
@IF(F235=1,$BS17*@EXP(-(FS206^2+FS214^2+$B223^2)/($GS1)),0)
G248:
@IF(G235=1,$BS17*@EXP(-(GS206^2+GS214^2+$B223^2)/($GS1)),0)
H248:
@IF(H235=1,$BS17*@EXP(-(HS206^2+HS214^2+$B223^2)/($GS1)),0)
A249: [W10] +FS27
B249:
@IF(B236=1,$BS17*@EXP(-(BS206^2+BS214^2+$B224^2)/($GS1)),0)
C249:
@IF(C236=1,$BS17*@EXP(-(CS206^2+CS214^2+$B224^2)/($GS1)),0)
D249:
@IF(D236=1,$BS17*@EXP(-(DS206^2+DS214^2+$B224^2)/($GS1)),0)
E249:
@IF(E236=1,$BS17*@EXP(-(ES206^2+ES214^2+$B224^2)/($GS1)),0)
F249:
@IF(F236=1,$BS17*@EXP(-(FS206^2+FS214^2+$B224^2)/($GS1)),0)
G249:
@IF(G236=1,$BS17*@EXP(-(GS206^2+GS214^2+$B224^2)/($GS1)),0)
H249:
@IF(H236=1,$BS17*@EXP(-(HS206^2+HS214^2+$B224^2)/($GS1)),0)
A250: [W10] +SGS27
B250:
@IF(B237=1,$BS17*@EXP(-(BS206^2+BS214^2+$B225^2)/($GS1)),0)
C250:
@IF(C237=1,$BS17*@EXP(-(CS206^2+CS214^2+$B225^2)/($GS1)),0)
D250:
@IF(D237=1,$BS17*@EXP(-(DS206^2+DS214^2+$B225^2)/($GS1)),0)
E250:
@IF(E237=1,$BS17*@EXP(-(ES206^2+ES214^2+$B225^2)/($GS1)),0)
F250:
@IF(F237=1,$BS17*@EXP(-(FS206^2+FS214^2+$B225^2)/($GS1)),0)
G250:
@IF(G237=1,$BS17*@EXP(-(GS206^2+GS214^2+$B225^2)/($GS1)),0)
H250:
@IF(H237=1,$BS17*@EXP(-(HS206^2+HS214^2+$B225^2)/($GS1)),0)
```

Right column:

```
A251: [W10] +SHS27
B251:
@IF(B238=1;$BS17*@EXP(-(BS206^2+BS226^2)/($GS1)),0)
C251:
@IF(C238=1,$BS17*@EXP(-(CS206^2+CS214^2+$B226^2)/($GS1)),0)
D251:
@IF(D238=1,$BS17*@EXP(-(DS206^2+DS214^2+$B226^2)/($GS1)),0)
E251:
@IF(E238=1,$BS17*@EXP(-(ES206^2+ES214^2+$B226^2)/($GS1)),0)
F251:
@IF(F238=1,$BS17*@EXP(-(FS206^2+FS214^2+$B226^2)/($GS1)),0)
G251:
@IF(G238=1,$BS17*@EXP(-(GS206^2+GS214^2+$B226^2)/($GS1)),0)
H251:
@IF(H238=1,$BS17*@EXP(-(HS206^2+HS214^2+$B226^2)/($GS1)),0)
A252: [W10] \-
B252: \-
C252: \-
D252: \-
E252: \-
F252: \-
G252: \-
H252: \-
A254: [W10] \-
B254: \-
A255: [W10] \-
B255: \-
A257: [W10] "X1(5)
B257: (2.3664077E-05*Q2)+G4
A260: [W10] "P1
A261: [W10] \-
B261: \-
A262: [W10] "P11
B262: ((B14*BB)+(B13*B9))*B257
A263: [W10] "P1(tot)
B263: +$BS27
C263: +$CS27
D263: +$ES27
E263: +$ES27
F263: +$FS27
G263: +$GS27
H263: +$HS27
B264: \-
C264: \-
D264: \-
E264: \-
F264: \-
G264: \-
H264: \-
B265: (1/$BS16)*($BS262-($BS12*$BS11*B$263))
C265: (1/$BS16)*($BS262-($BS12*$BS11*C$263))
D265: (1/$BS16)*($BS262-($BS12*$BS11*D$263))
E265: (1/$BS16)*($BS262-($BS12*$BS11*E$263))
```

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```
F265: (1/$B$16)*($B$262-($B$12*$B$11*F$263))
G265: (1/$B$16)*($B$262-($B$12*$B$11*G$263))
H265: (1/$B$16)*($B$262-($B$12*$B$11*H$263))
A267: [W10] 'P2
A268: [W10] \-
B268: \-
A269: [W10] "P2!
B269: +B12*B11*B257
B271: +$B$27
C271: +$C$27
D271: +$D$27
E271: +$E$27
F271: +$F$27
G271: +$G$27
H271: +$H$27
B272: \-
C272: \-
D272: \-
E272: \-
F272: \-
G272: \-
H272: \-
B273: (1/$B$16)*($B$269+($B$9*B$271))
C273: (1/$B$16)*($B$269+($B$9*C$271))
D273: (1/$B$16)*($B$269+($B$9*D$271))
E273: (1/$B$16)*($B$269+($B$9*E$271))
F273: (1/$B$16)*($B$269+($B$9*F$271))
G273: (1/$B$16)*($B$269+($B$9*G$271))
H273: (1/$B$16)*($B$269+($B$9*H$271))
A275: [W10] "P3
A276: [W10] \-
B276: \-
A279: [W10] +$B$27
B279: ($B$5/$B$9)*$A279
A280: [W10] +$C$27
B280: ($B$5/$B$9)*$A280
A281: [W10] +$D$27
B281: (,$B$5/$B$9)*$A281
A282: [W10] +$E$27
B282: ($B$5/$B$9)*$A292
A283: [W10] +$F$27
B283: ($B$5/$B$9)*$A283
A284: [W10] +$G$27
B284: ($B$5/$B$9)*$A284
A285: [W10] +$H$27
B285: ($B$5/$B$9)*$A285
A287: [W10] 'Logic table
A288: [W10] \-
B288: \-
B289: +$B$27
C289: +$C$27
D289: +$D$27
E289: +$E$27

F288: +$F$27
G289: +$G$27
H289: +$H$27
B290: \-
C290: \-
D290: \-
E290: \-
F290: \-
G290: \-
H290: \-
A291: [W10] +$B$27
B291: @IF(((B$265*2+B$273*2+$B$279*2)/($G$1))>=200,0,1)
C291: @IF(((C$265*2+C$273*2+$B$279*2)/($G$1))>=200,0,1)
D291: @IF(((D$265*2+D$273*2+$B$279*2)/($G$1))>=200,0,1)
E291: @IF(((E$265*2+E$273*2+$B$279*2)/($G$1))>=200,0,1)
F291: @IF(((F$265*2+F$273*2+$B$279*2)/($G$1))>=200,0,1)
G291: @IF(((G$265*2+G$273*2+$B$279*2)/($G$1))>=200,0,1)
H291: @IF(((H$265*2+H$273*2+$B$279*2)/($G$1))>=200,0,1)
A292: [W10] +$C$27
B292: @IF(((B$265*2+B$273*2+$B$280*2)/($G$1))>=200,0,1)
C292: @IF(((C$265*2+C$273*2+$B$280*2)/($G$1))>=200,0,1)
D292: @IF(((D$265*2+D$273*2+$B$280*2)/($G$1))>=200,0,1)
E292: @IF(((E$265*2+E$273*2+$B$280*2)/($G$1))>=200,0,1)
F292: @IF(((F$265*2+F$273*2+$B$280*2)/($G$1))>=200,0,1)
G292: @IF(((G$265*2+G$273*2+$B$280*2)/($G$1))>=200,0,1)
H292: @IF(((H$265*2+H$273*2+$B$280*2)/($G$1))>=200,0,1)
A293: [W10] +$D$27
B293: @IF(((B$265*2+B$273*2+$B$281*2)/($G$1))>=200,0,1)
C293: @IF(((C$265*2+C$273*2+$B$281*2)/($G$1))>=200,0,1)
D293: @IF(((D$265*2+D$273*2+$B$281*2)/($G$1))>=200,0,1)
E293: @IF(((E$265*2+E$273*2+$B$281*2)/($G$1))>=200,0,1)
F293: @IF(((F$265*2+F$273*2+$B$281*2)/($G$1))>=200,0,1)
G293: @IF(((G$265*2+G$273*2+$B$281*2)/($G$1))>=200,0,1)
H293: @IF(((H$265*2+H$273*2+$B$281*2)/($G$1))>=200,0,1)
A294: [W10] +$E$27
B294: @IF(((B$265*2+B$273*2+$B$282*2)/($G$1))>=200,0,1)
C294: @IF(((C$265*2+C$273*2+$B$282*2)/($G$1))>=200,0,1)
D294: @IF(((D$265*2+D$273*2+$B$282*2)/($G$1))>=200,0,1)
E294: @IF(((E$265*2+E$273*2+$B$282*2)/($G$1))>=200,0,1)
F294: @IF(((F$265*2+F$273*2+$B$282*2)/($G$1))>=200,0,1)
G294: @IF(((G$265*2+G$273*2+$B$282*2)/($G$1))>=200,0,1)
H294: @IF(((H$265*2+H$273*2+$B$282*2)/($G$1))>=200,0,1)
A295: [W10] +$F$27
B295: @IF(((B$265*2+B$273*2+$B$283*2)/($G$1))>=200,0,1)
C295: @IF(((C$265*2+C$273*2+$B$283*2)/($G$1))>=200,0,1)
D295: @IF(((D$265*2+D$273*2+$B$283*2)/($G$1))>=200,0,1)
E295: @IF(((E$265*2+E$273*2+$B$283*2)/($G$1))>=200,0,1)
F295: @IF(((F$265*2+F$273*2+$B$283*2)/($G$1))>=200,0,1)
G295: @IF(((G$265*2+G$273*2+$B$283*2)/($G$1))>=200,0,1)
H295: @IF(((H$265*2+H$273*2+$B$283*2)/($G$1))>=200,0,1)
A296: [W10] +$G$27
B296: @IF(((B$265*2+B$273*2+$B$284*2)/($G$1))>=200,0,1)
C296: @IF(((C$265*2+C$273*2+$B$284*2)/($G$1))>=200,0,1)
```

```
D296:   @IF(((D$265^2+D$273^2+$B$284^2)/(...$1))>-200,0,1)
E296:   @IF((((E$265^2+E$273^2+$B$284^2)/( .. $1))>-200,0,1)
F295:   @IF((((F$265^2+F$273^2+$B$284^2)/(..$C$1))>-200,0,1)
G295:   @IF((((G$265^2+G$273^2+$B$284^2)/($C$1))>-200,0,1)
H296:   @IF((((H$265^2+H$273^2+$B$284^2)/($G$1))>-200,0,1)
A297:   [W10] +H$27
B297:   @IF((((B$265^2+B$273^2+$B$285^2)/($G$1))>-200,0,1)
C297:   @IF((((C$265^2+C$273^2+$B$285^2)/($G$1))>-200,0,1)
D297:   @IF((((D$265^2+D$273^2+$B$285^2)/($G$1))>-200,0,1)
E297:   @IF((((E$265^2+E$273^2+$B$285^2)/($G$1))>-200,0,1)
F297:   @IF((((F$265^2+F$273^2+$B$285^2)/($G$1))>-200,0,1)
G297:   @IF((((G$265^2+G$273^2+$B$285^2)/($G$1))>-200,0,1)
H297:   @IF((((H$265^2+H$273^2+$B$285^2)/($G$1))>-200,0,1)
A300:   [W10] 'Rho
A301:   [W10] \-
B301:   \-
B302:   +B$27
C302:   +C$27
D302:   +D$27
E302:   +E$27
F302:   +F$27
G302:   +G$27
H302:   +H$27
B303:   \-
C303:   \-
D303:   \-
E303:   \-
F303:   \-
G303:   \-
H303:   \-
A304:   [W10] +B$27
B304:   @IF(B291-1,$B$17*@EXP(-(B$265^2+B$273^2+$B$279^2)/($G$1)),0)
C304:   @IF(C291-1,$B$17*@EXP(-(C$265^2+C$273^2+$B$279^2)/($G$1)),0)
D304:   @IF(D291-1,$B$17*@EXP(-(D$265^2+D$273^2+$B$279^2)/($G$1)),0)
E304:   @IF(E291-1,$B$17*@EXP(-(E$265^2+E$273^2+$B$279^2)/($G$1)),0)
F304:   @IF(F291-1,$B$17*@EXP(-(F$265^2+F$273^2+$B$279^2)/($G$1)),0)
G304:   @IF(G291-1,$B$17*@EXP(-(G$265^2+G$273^2+$B$279^2)/($G$1)),0)
H304:   @IF(H291-1,$B$17*@EXP(-(H$265^2+H$273^2+$B$279^2)/($G$1)),0)
A305:   [W10] +C$27
B305:   @IF(B292-1,$B$17*@EXP(-(B$265^2+B$273^2+$B$280^2)/($G$1)),0)
C305:   @IF(C292-1,$B$17*@EXP(-(C$265^2+C$273^2+$B$280^2)/($G$1)),0)
D305:   @IF(D292-1,$B$17*@EXP(-(D$265^2+D$273^2+$B$280^2)/($G$1)),0)
E305:
```

```
D296:   @IF((E292-1,$B$17*@EXP(-(E$265^2+E$273^2+$B$280^2)/($G$1)),0)
F305:   @IF(F292-1,$B$17*@EXP(-(F$265^2+F$273^2+$B$280^2)/($G$1)),0)
G305:   @IF(G292-1,$B$17*@EXP(-(G$265^2+G$273^2+$B$280^2)/($G$1)),0)
H305:   @IF(H292-1,$B$17*@EXP(-(H$265^2+H$273^2+$B$280^2)/($G$1)),0)
A306:   [W10] +D$27
B306:
C306:   @IF(B293-1,$B$17*@EXP(-(B$265^2+B$273^2+$B$281^2)/($G$1)),0)
D306:   @IF(C293-1,$B$17*@EXP(-(C$265^2+C$273^2+$B$281^2)/($G$1)),0)
D306:   @IF(D293-1,$B$17*@EXP(-(D$265^2+D$273^2+$B$281^2)/($G$1)),0)
E306:
F306:   @IF(E293-1,$B$17*@EXP(-(E$265^2+E$273^2+$B$281^2)/($G$1)),0)
F306:   @IF(F293-1,$B$17*@EXP(-(F$265^2+F$273^2+$B$281^2)/($G$1)),0)
G306:   @IF(G293-1,$B$17*@EXP(-(G$265^2+G$273^2+$B$281^2)/($G$1)),0)
H306:   @IF(H293-1,$B$17*@EXP(-(H$265^2+H$273^2+$B$281^2)/($G$1)),0)
A307:   [W10] +E$27
B307:
C307:   @IF(B294-1,$B$17*@EXP(-(B$265^2+B$273^2+$B$282^2)/($G$1)),0)
C307:   @IF(C294-1,$B$17*@EXP(-(C$265^2+C$273^2+$B$282^2)/($G$1)),0)
D307:   @IF(D294-1,$B$17*@EXP(-(D$265^2+D$273^2+$B$282^2)/($G$1)),0)
E307:   @IF(E294-1,$B$17*@EXP(-(E$265^2+E$273^2+$B$282^2)/($G$1)),0)
F307:   @IF(F294-1,$B$17*@EXP(-(F$265^2+F$273^2+$B$282^2)/($G$1)),0)
G307:   @IF(G294-1,$B$17*@EXP(-(G$265^2+G$273^2+$B$282^2)/($G$1)),0)
H307:   @IF(H294-1,$B$17*@EXP(-(H$265^2+H$273^2+$B$282^2)/($G$1)),0)
A308:   [W10] +F$27
B308:
C308:   @IF(B295-1,$B$17*@EXP(-(B$265^2+B$273^2+$B$283^2)/($G$1)),0)
D308:   @IF(C295-1,$B$17*@EXP(-(C$265^2+C$273^2+$B$283^2)/($G$1)),0)
D308:   @IF(D295-1,$B$17*@EXP(-(D$265^2+D$273^2+$B$283^2)/($G$1)),0)
E308:   @IF(E295-1,$B$17*@EXP(-(E$265^2+E$273^2+$B$283^2)/($G$1)),0)
F308:   @IF(F295-1,$B$17*@EXP(-(F$265^2+F$273^2+$B$283^2)/($G$1)),0)
G308:   @IF(G295-1,$B$17*@EXP(-(G$265^2+G$273^2+$B$283^2)/($G$1)),0)
H308:   @IF(H295-1,$B$17*@EXP(-(H$265^2+H$273^2+$B$283^2)/($G$1)),0)
A309:   [W10] +G$27
```

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```
A341:  [W10] +$E$27
B341:  (SB$5/$B$5)*$A341
A342:  [W10] +$F$27
B342:  (SB$5/$B$9)*$A342
A343:  [W10] +$G$27
B343:  (SB$5/$B$9)*$A343
A344:  (SB$5/$B$9)*$A344
A346:  [W10] 'Logic table
A347:  [W10] \-
B347:  \-
B348:  +$B$27
C348:  +$C$27
D348:  +$D$27
E348:  +$E$27
F348:  +$F$27
G348:  +$G$27
H348:  +$H$27
C349:  \-
D349:  \-
E349:  \-
F349:  \-
G349:  \-
H349:  \-
A350:  [W10] $B$27
B350:  @IF(((B$324^2+B$332^2+$B338^2)/($G$1))>=-200,0,1)
C350:  @IF(((C$324^2+C$332^2+$B338^2)/($G$1))>=-200,0,1)
D350:  @IF(((D$324^2+D$332^2+$B338^2)/($G$1))>=-200,0,1)
E350:  @IF(((E$324^2+E$332^2+$B338^2)/($G$1))>=-200,0,1)
F350:  @IF(((F$324^2+F$332^2+$B338^2)/($G$1))>=-200,0,1)
G350:  @IF(((G$324^2+G$332^2+$B338^2)/($G$1))>=-200,0,1)
H350:  @IF(((H$324^2+H$332^2+$B338^2)/($G$1))>=-200,0,1)
A351:  [W10] +$C$27
B351:  @IF(((B$324^2+B$332^2+$B339^2)/($G$1))>=-200,0,1)
C351:  @IF(((C$324^2+C$332^2+$B339^2)/($G$1))>=-200,0,1)
D351:  @IF(((D$324^2+D$332^2+$B339^2)/($G$1))>=-200,0,1)
F351:  @IF(((F$324^2+F$332^2+$B339^2)/($G$1))>=-200,0,1)
G351:  @IF(((G$324^2+G$332^2+$B339^2)/($G$1))>=-200,0,1)
H351:  @IF(((H$324^2+H$332^2+$B339^2)/($G$1))>=-200,0,1)
A352:  [W10] +$D$27
B352:  @IF(((B$324^2+B$332^2+$B340^2)/($G$1))>=-200,0,1)
C352:  @IF(((C$324^2+C$332^2+$B340^2)/($G$1))>=-200,0,1)
D352:  @IF(((D$324^2+D$332^2+$B340^2)/($G$1))>=-200,0,1)
E352:  @IF(((E$324^2+E$332^2+$B340^2)/($G$1))>=-200,0,1)
F352:  @IF(((F$324^2+F$332^2+$B340^2)/($G$1))>=-200,0,1)
G352:  @IF(((G$324^2+G$332^2+$B340^2)/($G$1))>=-200,0,1)
H352:  @IF(((H$324^2+H$332^2+$B340^2)/($G$1))>=-200,0,1)
A353:  [W10] +$E$27
B353:  @IF(((B$324^2+B$332^2+$B341^2)/($G$1))>=-200,0,1)
C353:  @IF(((C$324^2+C$332^2+$B341^2)/($G$1))>=-200,0,1)
D353:  @IF(((D$324^2+D$332^2+$B341^2)/($G$1))>=-200,0,1)

E353:  @IF(((E$324^2+E$332^2+$B341^2)/($G$1))>=-200,0,1)
F353:  @IF(((F$324^2+F$332^2+$B341^2)/($G$1))>=-200,0,1)
G353:  @IF(((G$324^2+G$332^2+$B341^2)/($G$1))>=-200,0,1)
H353:  @IF(((H$324^2+H$332^2+$B341^2)/($G$1))>=-200,0,1)
A354:  [W10] +$F$27
B354:  @IF(((B$324^2+B$332^2+$B342^2)/($G$1))>=-200,0,1)
C354:  @IF(((C$324^2+C$332^2+$B342^2)/($G$1))>=-200,0,1)
D354:  @IF(((D$324^2+D$332^2+$B342^2)/($G$1))>=-200,0,1)
E354:  @IF(((E$324^2+E$332^2+$B342^2)/($G$1))>=-200,0,1)
F354:  @IF(((F$324^2+F$332^2+$B342^2)/($G$1))>=-200,0,1)
G354:  @IF(((G$324^2+G$332^2+$B342^2)/($G$1))>=-200,0,1)
H354:  @IF(((H$324^2+H$332^2+$B342^2)/($G$1))>=-200,0,1)
A355:  [W10] +$G$27
B355:  @IF(((B$324^2+B$332^2+$B343^2)/($G$1))>=-200,0,1)
C355:  @IF(((C$324^2+C$332^2+$B343^2)/($G$1))>=-200,0,1)
D355:  @IF(((D$324^2+D$332^2+$B343^2)/($G$1))>=-200,0,1)
E355:  @IF(((E$324^2+E$332^2+$B343^2)/($G$1))>=-200,0,1)
F355:  @IF(((F$324^2+F$332^2+$B343^2)/($G$1))>=-200,0,1)
G355:  @IF(((G$324^2+G$332^2+$B343^2)/($G$1))>=-200,0,1)
H355:  @IF(((H$324^2+H$332^2+$B343^2)/($G$1))>=-200,0,1)
A356:  [W10] +$H$27
B356:  @IF(((B$324^2+B$332^2+$B344^2)/($G$1))>=-200,0,1)
C356:  @IF(((C$324^2+C$332^2+$B344^2)/($G$1))>=-200,0,1)
D356:  @IF(((D$324^2+D$332^2+$B344^2)/($G$1))>=-200,0,1)
E356:  @IF(((E$324^2+E$332^2+$B344^2)/($G$1))>=-200,0,1)
F356:  @IF(((F$324^2+F$332^2+$B344^2)/($G$1))>=-200,0,1)
G356:  @IF(((G$324^2+G$332^2+$B344^2)/($G$1))>=-200,0,1)
H356:  @IF(((H$324^2+H$332^2+$B344^2)/($G$1))>=-200,0,1)
A359:  [W10] 'Rho
A360:  [W10] \-
B360:  \-
B361:  +$B$27
C361:  +$C$27
D361:  +$D$27
E361:  +$E$27
F361:  +$F$27
G361:  +$G$27
H361:  +$H$27
B362:  \-
C362:  \-
D362:  \-
E362:  \-
F362:  \-
G362:  \-
H362:  \-
A363:  [W10] +$B$27
B363:  @IF(B$350=1,$B$17*@EXP(-(B$324^2+B$332^2+$B338^2)/($G$1)),0)
C363:  @IF(C$350=1,$B$17*@EXP(-(C$324^2+C$332^2+$B338^2)/($G$1)),0)
D363:  @IF(D$350=1,$B$17*@EXP(-(D$324^2+D$332^2+$B338^2)/($G$1)),0)
E363:
```

```
@IF(E350=1,$B$17*@EXP(-(C$324^2+E$332^2+$B338^2)/($G$1)),0)
F363:
@IF(F350=1,$B$17*@EXP(-(F$324^2+F$332^2+$B338^2)/($G$1)),0)
G363:
@IF(G350=1,$B$17*@EXP(-(G$324^2+G$332^2+$B338^2)/($G$1)),0)
H363:
@IF(H350=1,$B$17*@EXP(-(H$324^2+H$332^2+$B338^2)/($G$1)),0)
A364: [W10] +$C$27
B364:
@IF(B351=1,$B$17*@EXP(-(B$324^2+B$332^2+$B339^2)/($G$1)),0)
C364:
@IF(C351=1,$B$17*@EXP(-(C$324^2+C$332^2+$B339^2)/($G$1)),0)
D364:
@IF(D351=1,$B$17*@EXP(-(D$324^2+D$332^2+$B339^2)/($G$1)),0)
E364:
@IF(E351=1,$B$17*@EXP(-(E$324^2+E$332^2+$B339^2)/($G$1)),0)
F364:
@IF(F351=1,$B$17*@EXP(-(F$324^2+F$332^2+$B339^2)/($G$1)),0)
G364:
@IF(G351=1,$B$17*@EXP(-(G$324^2+G$332^2+$B339^2)/($G$1)),0)
H364:
@IF(H351=1,$B$17*@EXP(-(H$324^2+H$332^2+$B339^2)/($G$1)),0)
A365: [W10] +$D$27
B365:
@IF(B352=1,$B$17*@EXP(-(B$324^2+B$332^2+$B340^2)/($G$1)),0)
C365:
@IF(C352=1,$B$17*@EXP(-(C$324^2+C$332^2+$B340^2)/($G$1)),0)
D365:
@IF(D352=1,$B$17*@EXP(-(D$324^2+D$332^2+$B340^2)/($G$1)),0)
E365:
@IF(E352=1,$B$17*@EXP(-(E$324^2+E$332^2+$B340^2)/($G$1)),0)
F365:
@IF(F352=1,$B$17*@EXP(-(F$324^2+F$332^2+$B340^2)/($G$1)),0)
G365:
@IF(G352=1,$B$17*@EXP(-(G$324^2+G$332^2+$B340^2)/($G$1)),0)
H365:
@IF(H352=1,$B$17*@EXP(-(H$324^2+H$332^2+$B340^2)/($G$1)),0)
A366: [W10] +$E$27
B366:
@IF(B353=1,$B$17*@EXP(-(B$324^2+B$332^2+$B341^2)/($G$1)),0)
C366:
@IF(C353=1,$B$17*@EXP(-(C$324^2+C$332^2+$B341^2)/($G$1)),0)
D366:
@IF(D353=1,$B$17*@EXP(-(D$324^2+D$332^2+$B341^2)/($G$1)),0)
E366:
@IF(E353=1,$B$17*@EXP(-(E$324^2+E$332^2+$B341^2)/($G$1)),0)
F366:
@IF(F353=1,$B$17*@EXP(-(F$324^2+F$332^2+$B341^2)/($G$1)),0)
G366:
@IF(G353=1,$B$17*@EXP(-(G$324^2+G$332^2+$B341^2)/($G$1)),0)
H366:
@IF(H353=1,$B$17*@EXP(-(H$324^2+H$332^2+$B341^2)/($G$1)),0)
A367: [W10] +$F$27


B367:
@IF(B354=1,$B$17*@EXP(-(B$324^2+B$332^2+$B342^2)/($G$1)),0)
C367:
@IF(C354=1,$B$17*@EXP(-(C$324^2+C$332^2+$B342^2)/($G$1)),0)
D367:
@IF(D354=1,$B$17*@EXP(-(D$324^2+D$332^2+$B342^2)/($G$1)),0)
E367:
@IF(E354=1,$B$17*@EXP(-(E$324^2+E$332^2+$B342^2)/($G$1)),0)
F367:
@IF(F354=1,$B$17*@EXP(-(F$324^2+F$332^2+$B342^2)/($G$1)),0)
G367:
@IF(G354=1,$B$17*@EXP(-(G$324^2+G$332^2+$B342^2)/($G$1)),0)
H367:
@IF(H354=1,$B$17*@EXP(-(H$324^2+H$332^2+$B342^2)/($G$1)),0)
A368: [W10] +$G$27
B368:
@IF(B355=1,$B$17*@EXP(-(B$324^2+B$332^2+$B343^2)/($G$1)),0)
C368:
@IF(C355=1,$B$17*@EXP(-(C$324^2+C$332^2+$B343^2)/($G$1)),0)
D368:
@IF(D355=1,$B$17*@EXP(-(D$324^2+D$332^2+$B343^2)/($G$1)),0)
E368:
@IF(E355=1,$B$17*@EXP(-(E$324^2+E$332^2+$B343^2)/($G$1)),0)
F368:
@IF(F355=1,$B$17*@EXP(-(F$324^2+F$332^2+$B343^2)/($G$1)),0)
G368:
@IF(G355=1,$B$17*@EXP(-(G$324^2+G$332^2+$B343^2)/($G$1)),0)
H368:
@IF(H355=1,$B$17*@EXP(-(H$324^2+H$332^2+$B343^2)/($G$1)),0)
A369: [W10] +$H$27
B369:
@IF(B356=1,$B$17*@EXP(-(B$324^2+B$332^2+$B344^2)/($G$1)),0)
C369:
@IF(C356=1,$B$17*@EXP(-(C$324^2+C$332^2+$B344^2)/($G$1)),0)
D369:
@IF(D356=1,$B$17*@EXP(-(D$324^2+D$332^2+$B344^2)/($G$1)),0)
E369:
@IF(E356=1,$B$17*@EXP(-(E$324^2+E$332^2+$B344^2)/($G$1)),0)
F369:
@IF(F356=1,$B$17*@EXP(-(F$324^2+F$332^2+$B344^2)/($G$1)),0)
G369:
@IF(G356=1,$B$17*@EXP(-(G$324^2+G$332^2+$B344^2)/($G$1)),0)
H369:
@IF(H356=1,$B$17*@EXP(-(H$324^2+H$332^2+$B344^2)/($G$1)),0)
A370: [W10] \--
B370: \-
C370: \-
D370: \-
E370: \-
F370: \-
G370: \-
H370: \-
A372: [W10] \--
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```
B372:  \-
A373:  [W10] \-
B373:  \-
A375:  [W10] 'X1(7)
B375:  (7.099231E-05^02)+G4
A378:  [W10] 'P1
A379:  [W10] \-
B379:  \-
A380:  [W10] "P11
B380:  ((B14*B8)+(B13*B9))*B375
A381:  [W10] "P1(tot)
B381:  +SBS27
C381:  +SCS27
D381:  +SDS27
E381:  +SES27
F381:  +SFS27
G381:  +SGS27
H381:  +SHS27
B382:  \-
C382:  \-
D382:  \-
E382:  \-
F382:  \-
G382:  \-
H382:  (1/SBS16)*(SBS380-(SBS12*SBS11*BS381))
B383:  (1/SBS16)*(SBS380-(SBS12*SBS11*BS381))
C383:  (1/SBS16)*(SBS380-(SBS12*SBS11*CS381))
D383:  (1/SBS16)*(SBS380-(SBS12*SBS11*DS381))
E383:  (1/SBS16)*(SBS380-(SBS12*SBS11*ES381))
F383:  (1/SBS16)*(SBS380-(SBS12*SBS11*FS381))
G383:  (1/SBS16)*(SBS380-(SBS12*SBS11*GS381))
H383:  (1/SBS16)*(SBS380-(SBS12*SBS11*HS381))
A385:  [W10] 'P2
A386:  [W10] \-
A387:  [W10] "P21
B387:  +B12*B1*B375
A389:  [W10] 'P2(tot)
B389:  +SBS27
C389:  +SCS27
D389:  +SDS27
E389:  +SES27
F389:  +SFS27
G389:  +SGS27
H389:  +SHS27
B390:  \-
C390:  \-
D390:  \-
E390:  \-
F390:  \-
G390:  \-
H390:  \-
B391:  (1/SBS16)*(SBS387+(SBS9*BS389))

C391:  (1/SBS16)*(SBS387+(SBS9*CS389))
D391:  (1/SBS16)*(SBS387+(SBS9*DS389))
E391:  (1/SBS16)*(SBS387+(SBS9*ES389))
F391:  (1/SBS16)*(SBS387+(SBS9*FS389))
G391:  (1/SBS16)*(SBS387+(SBS9*GS389))
H391:  (1/SBS16)*(SBS387+(SBS9*HS389))
A393:  [W10] 'P3
A394:  [W10] \-
B394:  \-
A397:  [W10] +SBS27
B397:  (SBS5/SBS9)*SA397
A398:  [W10] +SCS27
B398:  (SBS5/SBS9)*SA398
A399:  [W10] +SDS27
B399:  (SBS5/SBS9)*SA399
A400:  [W10] +SES27
B400:  (SBS5/SBS9)*SA400
A401:  [W10] +SFS27
B401:  (SBS5/SBS9)*SA401
A402:  [W10] +SGS27
B402:  (SBS5/SBS9)*SA402
A403:  [W10] +SHS27
B403:  (SBS5/SBS9)*SA403
A405:  [W10] 'Logic table
A406:  [W10] \-
B406:  \-
B407:  +SBS27
C407:  +SCS27
D407:  +SDS27
E407:  +SES27
F407:  +SFS27
G407:  +SGS27
H407:  +SHS27
B408:  \-
C408:  \-
D408:  \-
E408:  \-
F408:  \-
G408:  \-
H408:  \-
A409:  [W10] +SBS27
B409:  @IF(((BS383^2+BS391^2+BS397^2)/(SGS1))>=200,0,1)
C409:  @IF(((CS383^2+CS391^2+CS397^2)/(SGS1))>=200,0,1)
D409:  @IF(((DS383^2+DS391^2+DS397^2)/(SGS1))>=200,0,1)
E409:  @IF(((ES383^2+ES391^2+ES397^2)/(SGS1))>=200,0,1)
F409:  @IF(((FS383^2+FS391^2+FS397^2)/(SGS1))>=200,0,1)
G409:  @IF(((GS393^2+GS391^2+GS397^2)/(SGS1))>=200,0,1)
H409:  @IF(((HS383^2+HS391^2+HS397^2)/(SGS1))>=200,0,1)
A410:  [W10] +SCS27
B410:  @IF(((BS383^2+BS390^2)/(SGS1))>=200,0,1)
C410:  @IF(((CS383^2+CS390^2)/(SGS1))>=200,0,1)
D410:  @IF(((DS383^2+DS390^2)/(SGS1))>=200,0,1)
E410:  @IF(((ES383^2+ES390^2)/(SGS1))>=200,0,1)
```

```
FW10:  @IF(((F$383^2+F$391^2+$B$398^2)/($G$1))>=200,0,1)
GW10:  @IF(((G$383^2+G$391^2+$B$398^2)/($G$1))>=200,0,1)
HW10:  @IF(((H$383^2+H$391^2+$B$398^2)/($G$1))>=200,0,1)
AW11:  [W10] +$D$27
BW11:  @IF(((B$383^2+B$391^2+$B$399^2)/($G$1))>=200,0,1)
CW11:  @IF(((C$383^2+C$391^2+$B$399^2)/($G$1))>=200,0,1)
DW11:  @IF(((D$383^2+D$391^2+$B$399^2)/($G$1))>=200,0,1)
EW11:  @IF(((E$383^2+E$391^2+$B$399^2)/($G$1))>=200,0,1)
FW11:  @IF(((F$383^2+F$391^2+$B$399^2)/($G$1))>=200,0,1)
GW11:  @IF(((G$383^2+G$391^2+$B$399^2)/($G$1))>=200,0,1)
HW11:  @IF(((H$383^2+H$391^2+$B$399^2)/($G$1))>=200,0,1)
AW12:  [W10] +$E$27
BW12:  @IF(((C$383^2+B$391^2+$B$400^2)/($G$1))>=200,0,1)
CW12:  @IF(((C$383^2+C$391^2+$B$400^2)/($G$1))>=200,0,1)
DW12:  @IF(((D$383^2+D$391^2+$B$400^2)/($G$1))>=200,0,1)
EW12:  @IF(((E$383^2+E$391^2+$B$400^2)/($G$1))>=200,0,1)
FW12:  @IF(((F$383^2+F$391^2+$B$400^2)/($G$1))>=200,0,1)
GW12:  @IF(((G$383^2+G$391^2+$B$400^2)/($G$1))>=200,0,1)
HW12:  @IF(((H$383^2+H$391^2+$B$400^2)/($G$1))>=200,0,1)
AW13:  [W10] +$F$27
BW13:  @IF(((B$383^2+B$391^2+$B$401^2)/($G$1))>=200,0,1)
CW13:  @IF(((C$383^2+C$391^2+$B$401^2)/($G$1))>=200,0,1)
DW13:  @IF(((D$383^2+D$391^2+$B$401^2)/($G$1))>=200,0,1)
EW13:  @IF(((E$383^2+E$391^2+$B$401^2)/($G$1))>=200,0,1)
FW13:  @IF(((F$383^2+F$391^2+$B$401^2)/($G$1))>=200,0,1)
GW13:  @IF(((G$383^2+G$391^2+$B$401^2)/($G$1))>=200,0,1)
HW13:  @IF(((H$383^2+H$391^2+$B$401^2)/($G$1))>=200,0,1)
AW14:  [W10] +$G$27
BW14:  @IF(((B$383^2+B$391^2+$B$402^2)/($G$1))>=200,0,1)
CW14:  @IF(((C$383^2+C$391^2+$B$402^2)/($G$1))>=200,0,1)
DW14:  @IF(((D$383^2+D$391^2+$B$402^2)/($G$1))>=200,0,1)
EW14:  @IF(((E$383^2+E$391^2+$B$402^2)/($G$1))>=200,0,1)
FW14:  @IF(((F$383^2+F$391^2+$B$402^2)/($G$1))>=200,0,1)
GW14:  @IF(((G$383^2+G$391^2+$B$402^2)/($G$1))>=200,0,1)
HW14:  @IF(((H$383^2+H$391^2+$B$402^2)/($G$1))>=200,0,1)
AW15:  [W10] +$H$27
BW15:  @IF(((B$383^2+B$391^2+$B$403^2)/($G$1))>=200,0,1)
CW15:  @IF(((C$383^2+C$391^2+$B$403^2)/($G$1))>=200,0,1)
DW15:  @IF(((D$393^2+D$391^2+$B$403^2)/($G$1))>=200,0,1)
EW15:  @IF(((E$383^2+E$391^2+$B$403^2)/($G$1))>=200,0,1)
FW15:  @IF(((F$383^2+F$391^2+$B$403^2)/($G$1))>=200,0,1)
GW15:  @IF(((G$383^2+G$391^2+$B$403^2)/($G$1))>=200,0,1)
HW15:  @IF(((H$383^2+H$391^2+$B$403^2)/($G$1))>=200,0,1)
AW18:  [W10]  'Rho
AW19:  [W10] /-
BW19:  /-
BW20:  +$B$27
CW20:  +$C$27
DW20:  +$D$27
EW20:  +$E$27
FW20:  +$F$27
GW20:  +$G$27
HW20:  +$H$27
```

```
BW21:  /-
CW21:  /-
DW21:  /-
EW21:  /-
FW21:  /-
GW21:  /-
HW21:  /-
AW22:  [W10] +$B$27
BW22:  @IF(B409=1,$B$17*@EXP(-(B$383^2+$B$397^2)/($G$1)),0)
CW22:  @IF(C409=1,$B$17*@EXP(-(C$383^2+$C$397^2)/($G$1)),0)
DW22:  @IF(D409=1,$B$17*@EXP(-(D$383^2+$D$397^2)/($G$1)),0)
EW22:  @IF(E408=1,$B$17*@EXP(-(E$383^2+$E$397^2)/($G$1)),0)
FW22:  @IF(F409=1,$B$17*@EXP(-(F$383^2+$F$397^2)/($G$1)),0)
GW22:  @IF(G408=1,$B$17*@EXP(-(G$383^2+$G$397^2)/($G$1)),0)
HW22:  @IF(H409=1,$B$17*@EXP(-(H$383^2+$H$397^2)/($G$1)),0)
AW23:  [W10] +$C$27
BW23:  @IF(B410=1,$B$17*@EXP(-(B$383^2+$B$398^2)/($G$1)),0)
CW23:  @IF(C410=1,$B$17*@EXP(-(C$383^2+$C$398^2)/($G$1)),0)
DW23:  @IF(D410=1,$B$17*@EXP(-(D$383^2+$D$398^2)/($G$1)),0)
EW23:  @IF(E410=1,$B$17*@EXP(-(E$383^2+$E$398^2)/($G$1)),0)
FW23:  @IF(F410=1,$B$17*@EXP(-(F$383^2+$F$398^2)/($G$1)),0)
GW23:  @IF(G410=1,$B$17*@EXP(-(G$383^2+$G$398^2)/($G$1)),0)
HW23:  @IF(H410=1,$B$17*@EXP(-(H$383^2+$H$398^2)/($G$1)),0)
AW24:  [W10] +$D$27
BW24:  @IF(B411=1,$B$17*@EXP(-(B$383^2+$B$399^2)/($G$1)),0)
CW24:  @IF(C411=1,$B$17*@EXP(-(C$383^2+$C$399^2)/($G$1)),0)
DW24:  @IF(D411=1,$B$17*@EXP(-(D$383^2+$D$399^2)/($G$1)),0)
EW24:  @IF(E411=1,$B$17*@EXP(-(E$383^2+$E$399^2)/($G$1)),0)
FW24:  @IF(F411=1,$B$17*@EXP(-(F$383^2+$F$399^2)/($G$1)),0)
GW24:  @IF(G411=1,$B$17*@EXP(-(G$383^2+$G$399^2)/($G$1)),0)
HW24:  @IF(H411=1,$B$17*@EXP(-(H$383^2+$H$399^2)/($G$1)),0)
AW25:  [W10] +$E$27
```

```
B425:
@IF(B412=1,$B$17*@EXP(-(B$383^2+B$391^2+$B$400^2)/($G$1)),0)
C425:
@IF(C412=1,$B$17*@EXP(-(C$383^2+C$391^2+$B$400^2)/($G$1)),0)
D425:
@IF(D412=1,$B$17*@EXP(-(D$383^2+D$391^2+$B$400^2)/($G$1)),0)
E425:
@IF(E412=1,$B$17*@EXP(-(E$383^2+E$391^2+$B$400^2)/($G$1)),0)
F425:
@IF(F412=1,$B$17*@EXP(-(F$383^2+F$391^2+$B$400^2)/($G$1)),0)
G425:
@IF(G412=1,$B$17*@EXP(-(G$383^2+G$391^2+$B$400^2)/($G$1)),0)
H425:
@IF(H412=1,$B$17*@EXP(-(H$383^2+H$391^2+$B$400^2)/($G$1)),0)
A426: [W10] +$F$27
B426:
@IF(B413=1,$B$17*@EXP(-(B$383^2+B$391^2+$B$401^2)/($G$1)),0)
C426:
@IF(C413=1,$B$17*@EXP(-(C$383^2+C$391^2+$B$401^2)/($G$1)),0)
D426:
@IF(D413=1,$B$17*@EXP(-(D$383^2+D$391^2+$B$401^2)/($G$1)),0)
E426:
@IF(E413=1,$B$17*@EXP(-(E$383^2+E$391^2+$B$401^2)/($G$1)),0)
F426:
@IF(F413=1,$B$17*@EXP(-(F$383^2+F$391^2+$B$401^2)/($G$1)),0)
G426:
@IF(G413=1,$B$17*@EXP(-(G$383^2+G$391^2+$B$401^2)/($G$1)),0)
H426:
@IF(H413=1,$B$17*@EXP(-(H$383^2+H$391^2+$B$401^2)/($G$1)),0)
A427: [W10] +$G$27
B427:
@IF(B414=1,$B$17*@EXP(-(B$383^2+B$391^2+$B$402^2)/($G$1)),0)
C427:
@IF(C414=1,$B$17*@EXP(-(C$383^2+C$391^2+$B$402^2)/($G$1)),0)
D427:
@IF(D414=1,$B$17*@EXP(-(D$383^2+D$391^2+$B$402^2)/($G$1)),0)
E427:
@IF(E414=1,$B$17*@EXP(-(E$383^2+E$391^2+$B$402^2)/($G$1)),0)
F427:
@IF(F414=1,$B$17*@EXP(-(F$383^2+F$391^2+$B$402^2)/($G$1)),0)
G427:
@IF(G414=1,$B$17*@EXP(-(G$383^2+G$391^2+$B$402^2)/($G$1)),0)
H427:
@IF(H414=1,$B$17*@EXP(-(H$383^2+H$391^2+$B$402^2)/($G$1)),0)
A428: [W10] +$H$27
B428:
@IF(B415=1,$B$17*@EXP(-(B$383^2+B$391^2+$B$403^2)/($G$1)),0)
C428:
@IF(C415=1,$B$17*@EXP(-(C$383^2+C$391^2+$B$403^2)/($G$1)),0)
D428:
@IF(D415=1,$B$17*@EXP(-(D$383^2+D$391^2+$B$403^2)/($G$1)),0)
E428:
@IF(E415=1,$B$17*@EXP(-(E$383^2+E$391^2+$B$403^2)/($G$1)),0)
F428:
@IF(F415=1,$B$17*@EXP(-(F$383^2+F$391^2+$B$403^2)/($G$1)),0)
G428:
@IF(G415=1,$B$17*@EXP(-(G$383^2+G$391^2+$B$403^2)/($G$1)),0)
H428:
@IF(H415=1,$B$17*@EXP(-(H$383^2+H$391^2+$B$403^2)/($G$1)),0)
A429: [W10] \-
B429: \-
C429: \-
D429: \-
E429: \-
F429: \-
G429: \-
H429: \-
A433: [W10] 'Total Rhosum
A434: [W10] +I76+I135+I194+I253+I312+I371+I430
A436: [W10] +G2
B436: 'm/s vel
A437: [W10] +D2
B437: 'scale
A438: [W10] +D1
B438: 'sec
A439: [W10] +D3
B439: 'particles
A441: [W10] 'Part XI(1)
A442: [W10] \-
B443: @INT((B68/$A$434)*$D$3)
C443: @INT((C68/$A$434)*$D$3)
D443: @INT((D68/$A$434)*$D$3)
E443: @INT((E68/$A$434)*$D$3)
F443: @INT((F68/$A$434)*$D$3)
G443: @INT((G68/$A$434)*$D$3)
H443: @INT((H68/$A$434)*$D$3)
B444: @INT((B69/$A$434)*$D$3)
C444: @INT((C69/$A$434)*$D$3)
D444: @INT((D69/$A$434)*$D$3)
E444: @INT((E69/$A$434)*$D$3)
F444: @INT((F69/$A$434)*$D$3)
G444: @INT((G69/$A$434)*$D$3)
H444: @INT((H69/$A$434)*$D$3)
B445: @INT((B70/$A$434)*$D$3)
C445: @INT((C70/$A$434)*$D$3)
D445: @INT((D70/$A$434)*$D$3)
E445: @INT((E70/$A$434)*$D$3)
F445: @INT((F70/$A$434)*$D$3)
G445: @INT((G70/$A$434)*$D$3)
H445: @INT((H70/$A$434)*$D$3)
B446: @INT((B71/$A$434)*$D$3)
C446: @INT((C71/$A$434)*$D$3)
D446: @INT((D71/$A$434)*$D$3)
E446: @INT((E71/$A$434)*$D$3)
F446: @INT((F71/$A$434)*$D$3)
G446: @INT((G71/$A$434)*$D$3)
```

```
H446:  @INT((H71/$A$434)*$D$3)
B447:  @INT((B72/$A$434)*$D$3)
C447:  @INT((C72/$A$434)*$D$3)
D447:  @INT((D72/$A$434)*$D$3)
E447:  @INT((E72/$A$434)*$D$3)
F447:  @INT((F72/$A$434)*$D$3)
G447:  @INT((G72/$A$434)*$D$3)
H447:  @INT((H72/$A$434)*$D$3)
B448:  @INT((B73/$A$434)*$D$3)
C448:  @INT((C73/$A$434)*$D$3)
D448:  @INT((D73/$A$434)*$D$3)
E448:  @INT((E73/$A$434)*$D$3)
F448:  @INT((F73/$A$434)*$D$3)
G448:  @INT((G73/$A$434)*$D$3)
H448:  @INT((H73/$A$434)*$D$3)
B449:  @INT((B74/$A$434)*$D$3)
C449:  @INT((C74/$A$434)*$D$3)
D449:  @INT((D74/$A$434)*$D$3)
E449:  @INT((E74/$A$434)*$D$3)
F449:  @INT((F74/$A$434)*$D$3)
G449:  @INT((G74/$A$434)*$D$3)
H449:  @INT((H74/$A$434)*$D$3)
A451:  [W10] \-
A452:  [W10] 'Part X1(2)
B453:  @INT((B127/$A$434)*$D$3)
C453:  @INT((C127/$A$434)*$D$3)
D453:  @INT((D127/$A$434)*$D$3)
E453:  @INT((E127/$A$434)*$D$3)
F453:  @INT((F127/$A$434)*$D$3)
G453:  @INT((G127/$A$434)*$D$3)
H453:  @INT((H127/$A$434)*$D$3)
B454:  @INT((B128/$A$434)*$D$3)
C454:  @INT((C128/$A$434)*$D$3)
D454:  @INT((D128/$A$434)*$D$3)
E454:  @INT((E128/$A$434)*$D$3)
F454:  @INT((F128/$A$434)*$D$3)
G454:  @INT((G128/$A$434)*$D$3)
H454:  @INT((H128/$A$434)*$D$3)
B455:  @INT((B129/$A$434)*$D$3)
C455:  @INT((C129/$A$434)*$D$3)
D455:  @INT((D129/$A$434)*$D$3)
E455:  @INT((E129/$A$434)*$D$3)
F455:  @INT((F129/$A$434)*$D$3)
G455:  @INT((G129/$A$434)*$D$3)
H455:  @INT((H129/$A$434)*$D$3)
B456:  @INT((B130/$A$434)*$D$3)
C456:  @INT((C130/$A$434)*$D$3)
D456:  @INT((D130/$A$434)*$D$3)
E456:  @INT((E130/$A$434)*$D$3)
F456:  @INT((F130/$A$434)*$D$3)
G456:  @INT((G130/$A$434)*$D$3)
H456:  @INT((H130/$A$434)*$D$3)
B457:  @INT((B131/$A$434)*$D$3)

C457:  @INT((C131/$A$434)*$D$3)
D457:  @INT((D131/$A$434)*$D$3)
E457:  @INT((E131/$A$434)*$D$3)
F457:  @INT((F131/$A$434)*$D$3)
G457:  @INT((G131/$A$434)*$D$3)
H457:  @INT((H131/$A$434)*$D$3)
B458:  @INT((B132/$A$434)*$D$3)
C458:  @INT((C132/$A$434)*$D$3)
D458:  @INT((D132/$A$434)*$D$3)
E458:  @INT((E132/$A$434)*$D$3)
F458:  @INT((F132/$A$434)*$D$3)
G458:  @INT((G132/$A$434)*$D$3)
H458:  @INT((H132/$A$434)*$D$3)
B459:  @INT((B133/$A$434)*$D$3)
C459:  @INT((C133/$A$434)*$D$3)
D459:  @INT((D133/$A$434)*$D$3)
E459:  @INT((E133/$A$434)*$D$3)
F459:  @INT((F133/$A$434)*$D$3)
G459:  @INT((G133/$A$434)*$D$3)
H459:  @INT((H133/$A$434)*$D$3)
A461:  [W10] 'Part X1(3)
A462:  [W10] \-
B463:  @INT((B185/$A$434)*$D$3)
C463:  @INT((C186/$A$434)*$D$3)
D463:  @INT((D186/$A$434)*$D$3)
E463:  @INT((E186/$A$434)*$D$3)
F463:  @INT((F186/$A$434)*$D$3)
G463:  @INT((G186/$A$434)*$D$3)
H463:  @INT((H186/$A$434)*$D$3)
B464:  @INT((B187/$A$434)*$D$3)
C464:  @INT((C187/$A$434)*$D$3)
D464:  @INT((D187/$A$434)*$D$3)
E464:  @INT((E187/$A$434)*$D$3)
F464:  @INT((F187/$A$434)*$D$3)
G464:  @INT((G187/$A$434)*$D$3)
H464:  @INT((H187/$A$434)*$D$3)
B465:  @INT((B188/$A$434)*$D$3)
C465:  @INT((C188/$A$434)*$D$3)
D465:  @INT((D188/$A$434)*$D$3)
E465:  @INT((E188/$A$434)*$D$3)
F465:  @INT((F188/$A$434)*$D$3)
G465:  @INT((G188/$A$434)*$D$3)
H465:  @INT((H188/$A$434)*$D$3)
B466:  @INT((B189/$A$434)*$D$3)
C466:  @INT((C189/$A$434)*$D$3)
D466:  @INT((D189/$A$434)*$D$3)
E466:  @INT((E189/$A$434)*$D$3)
F466:  @INT((F189/$A$434)*$D$3)
G466:  @INT((G189/$A$434)*$D$3)
H466:  @INT((H189/$A$434)*$D$3)
B467:  @INT((B190/$A$434)*$D$3)
C467:  @INT((C190/$A$434)*$D$3)
D467:  @INT((D190/$A$434)*$D$3)
```

E467:   @INT((E190/$A$434)*$D$3)
F467:   @INT((F190/$A$434)*$D$3)
G467:   @INT((G190/$A$434)*$D$3)
H467:   @INT((H190/$A$434)*$D$3)
B468:   @INT((B191/$A$434)*$D$3)
C468:   @INT((C191/$A$434)*$D$3)
D468:   @INT((D191/$A$434)*$D$3)
E468:   @INT((E191/$A$434)*$D$3)
F468:   @INT((F191/$A$434)*$D$3)
G468:   @INT((G191/$A$434)*$D$3)
H468:   @INT((H191/$A$434)*$D$3)
B469:   @INT((B192/$A$434)*$D$3)
C469:   @INT((C192/$A$434)*$D$3)
D469:   @INT((D192/$A$434)*$D$3)
E469:   @INT((E192/$A$434)*$D$3)
F469:   @INT((F192/$A$434)*$D$3)
G469:   @INT((G192/$A$434)*$D$3)
H469:   @INT((H192/$A$434)*$D$3)
A471:   [W10] 'Part X1(4)
A472:   [W10] \-
B473:   @INT((B245/$A$434)*$D$3)
C473:   @INT((C245/$A$434)*$D$3)
D473:   @INT((D245/$A$434)*$D$3)
E473:   @INT((E245/$A$434)*$D$3)
F473:   @INT((F245/$A$434)*$D$3)
G473:   @INT((G245/$A$434)*$D$3)
H473:   @INT((H245/$A$434)*$D$3)
B474:   @INT((B246/$A$434)*$D$3)
C474:   @INT((C246/$A$434)*$D$3)
D474:   @INT((D246/$A$434)*$D$3)
E474:   @INT((E246/$A$434)*$D$3)
F474:   @INT((F246/$A$434)*$D$3)
G474:   @INT((G246/$A$434)*$D$3)
H474:   @INT((H246/$A$434)*$D$3)
B475:   @INT((B247/$A$434)*$D$3)
C475:   @INT((C247/$A$434)*$D$3)
D475:   @INT((D247/$A$434)*$D$3)
E475:   @INT((E247/$A$434)*$D$3)
F475:   @INT((F247/$A$434)*$D$3)
G475:   @INT((G247/$A$434)*$D$3)
H475:   @INT((H247/$A$434)*$D$3)
B476:   @INT((B248/$A$434)*$D$3)
C476:   @INT((C248/$A$434)*$D$3)
D476:   @INT((D248/$A$434)*$D$3)
E476:   @INT((E248/$A$434)*$D$3)
F476:   @INT((F248/$A$434)*$D$3)
G476:   @INT((G248/$A$434)*$D$3)
H476:   @INT((H248/$A$434)*$D$3)
B477:   @INT((B249/$A$434)*$D$3)
C477:   @INT((C249/$A$434)*$D$3)
D477:   @INT((D249/$A$434)*$D$3)
E477:   @INT((E249/$A$434)*$D$3)
F477:   @INT((F249/$A$434)*$D$3)

G477:   @INT((G249/$A$434)*$D$3)
H477:   @INT((H249/$A$434)*$D$3)
B478:   @INT((B250/$A$434)*$D$3)
C478:   @INT((C250/$A$434)*$D$3)
D478:   @INT((D250/$A$434)*$D$3)
E478:   @INT((E250/$A$434)*$D$3)
F478:   @INT((F250/$A$434)*$D$3)
G478:   @INT((G250/$A$434)*$D$3)
H478:   @INT((H250/$A$434)*$D$3)
B479:   @INT((B251/$A$434)*$D$3)
C479:   @INT((C251/$A$434)*$D$3)
D479:   @INT((D251/$A$434)*$D$3)
E479:   @INT((E251/$A$434)*$D$3)
F479:   @INT((F251/$A$434)*$D$3)
G479:   @INT((G251/$A$434)*$D$3)
H479:   @INT((H251/$A$434)*$D$3)
A481:   [W10] 'Part X1(5)
A482:   [W10] \-
B483:   @INT((B304/$A$434)*$D$3)
C483:   @INT((C304/$A$434)*$D$3)
D483:   @INT((D304/$A$434)*$D$3)
E483:   @INT((E304/$A$434)*$D$3)
F483:   @INT((F304/$A$434)*$D$3)
G483:   @INT((G304/$A$434)*$D$3)
H483:   @INT((H304/$A$434)*$D$3)
B484:   @INT((B305/$A$434)*$D$3)
C484:   @INT((C305/$A$434)*$D$3)
D484:   @INT((D305/$A$434)*$D$3)
E484:   @INT((E305/$A$434)*$D$3)
F484:   @INT((F305/$A$434)*$D$3)
G484:   @INT((G305/$A$434)*$D$3)
H484:   @INT((H305/$A$434)*$D$3)
B485:   @INT((B306/$A$434)*$D$3)
C485:   @INT((C306/$A$434)*$D$3)
D485:   @INT((D306/$A$434)*$D$3)
E485:   @INT((E306/$A$434)*$D$3)
F485:   @INT((F306/$A$434)*$D$3)
G485:   @INT((G306/$A$434)*$D$3)
H485:   @INT((H306/$A$434)*$D$3)
B486:   @INT((B307/$A$434)*$D$3)
C486:   @INT((C307/$A$434)*$D$3)
D486:   @INT((D307/$A$434)*$D$3)
E486:   @INT((E307/$A$434)*$D$3)
F486:   @INT((F307/$A$434)*$D$3)
G486:   @INT((G307/$A$434)*$D$3)
H486:   @INT((H307/$A$434)*$D$3)
B487:   @INT((B308/$A$434)*$D$3)
C487:   @INT((C308/$A$434)*$D$3)
D487:   @INT((D308/$A$434)*$D$3)
E487:   @INT((E308/$A$434)*$D$3)
F487:   @INT((F308/$A$434)*$D$3)
G487:   @INT((G308/$A$434)*$D$3)
H487:   @INT((H308/$A$434)*$D$3)

```
B488:  @INT((B309/$A$434)*$D$3)
C488:  @INT((C309/$A$434)*$D$3)
D488:  @INT((D309/$A$434)*$D$3)
E488:  @INT((E309/$A$434)*$D$3)
F488:  @INT((F309/$A$434)*$D$3)
G488:  @INT((G309/$A$434)*$D$3)
H488:  @INT((H309/$A$434)*$D$3)
B489:  @INT((B310/$A$434)*$D$3)
C489:  @INT((C310/$A$434)*$D$3)
D489:  @INT((D310/$A$434)*$D$3)
E489:  @INT((E310/$A$434)*$D$3)
F489:  @INT((F310/$A$434)*$D$3)
G489:  @INT((G310/$A$434)*$D$3)
H489:  @INT((H310/$A$434)*$D$3)
A491:  [W10] '\-
A492:  [W10] 'Part X1(6)
C493:  @INT((B363/$A$434)*$D$3)
D493:  @INT((C363/$A$434)*$D$3)
E493:  @INT((D363/$A$434)*$D$3)
F493:  @INT((E363/$A$434)*$D$3)
G493:  @INT((F363/$A$434)*$D$3)
H493:  @INT((G363/$A$434)*$D$3)
B494:  @INT((H363/$A$434)*$D$3)
C494:  @INT((B364/$A$434)*$D$3)
D494:  @INT((C364/$A$434)*$D$3)
E494:  @INT((D364/$A$434)*$D$3)
F494:  @INT((E364/$A$434)*$D$3)
G494:  @INT((F364/$A$434)*$D$3)
H494:  @INT((G364/$A$434)*$D$3)
B495:  @INT((H364/$A$434)*$D$3)
C495:  @INT((B365/$A$434)*$D$3)
D495:  @INT((C365/$A$434)*$D$3)
E495:  @INT((D365/$A$434)*$D$3)
F495:  @INT((E365/$A$434)*$D$3)
G495:  @INT((F365/$A$434)*$D$3)
H495:  @INT((G365/$A$434)*$D$3)
B496:  @INT((H365/$A$434)*$D$3)
C496:  @INT((B366/$A$434)*$D$3)
D496:  @INT((C366/$A$434)*$D$3)
E496:  @INT((D366/$A$434)*$D$3)
F496:  @INT((E366/$A$434)*$D$3)
G496:  @INT((F366/$A$434)*$D$3)
H496:  @INT((G366/$A$434)*$D$3)
B497:  @INT((H366/$A$434)*$D$3)
C497:  @INT((B367/$A$434)*$D$3)
D497:  @INT((C367/$A$434)*$D$3)
E497:  @INT((D367/$A$434)*$D$3)
F497:  @INT((E367/$A$434)*$D$3)
G497:  @INT((F367/$A$434)*$D$3)
H497:  @INT((G367/$A$434)*$D$3)
B498:  @INT((H367/$A$434)*$D$3)
C498:  @INT((C368/$A$434)*$D$3)

D498:  @INT((D368/$A$434)*$D$3)
E498:  @INT((E368/$A$434)*$D$3)
F498:  @INT((F368/$A$434)*$D$3)
G498:  @INT((G368/$A$434)*$D$3)
H498:  @INT((H368/$A$434)*$D$3)
B499:  @INT((B369/$A$434)*$D$3)
C499:  @INT((C369/$A$434)*$D$3)
D499:  @INT((D369/$A$434)*$D$3)
E499:  @INT((E369/$A$434)*$D$3)
F499:  @INT((F369/$A$434)*$D$3)
G499:  @INT((G369/$A$434)*$D$3)
H499:  @INT((H369/$A$434)*$D$3)
A501:  [W10] 'Part X1(7)
A502:  [W10] '\-
B503:  @INT((B422/$A$434)*$D$3)
C503:  @INT((C422/$A$434)*$D$3)
D503:  @INT((D422/$A$434)*$D$3)
E503:  @INT((E422/$A$434)*$D$3)
F503:  @INT((F422/$A$434)*$D$3)
G503:  @INT((G422/$A$434)*$D$3)
H503:  @INT((H422/$A$434)*$D$3)
B504:  @INT((B423/$A$434)*$D$3)
C504:  @INT((C423/$A$434)*$D$3)
D504:  @INT((D423/$A$434)*$D$3)
E504:  @INT((E423/$A$434)*$D$3)
F504:  @INT((F423/$A$434)*$D$3)
G504:  @INT((G423/$A$434)*$D$3)
H504:  @INT((H423/$A$434)*$D$3)
B505:  @INT((B424/$A$434)*$D$3)
C505:  @INT((C424/$A$434)*$D$3)
D505:  @INT((D424/$A$434)*$D$3)
E505:  @INT((E424/$A$434)*$D$3)
F505:  @INT((F424/$A$434)*$D$3)
G505:  @INT((G424/$A$434)*$D$3)
H505:  @INT((H424/$A$434)*$D$3)
B506:  @INT((B425/$A$434)*$D$3)
C506:  @INT((C425/$A$434)*$D$3)
D506:  @INT((D425/$A$434)*$D$3)
E506:  @INT((E425/$A$434)*$D$3)
F506:  @INT((F425/$A$434)*$D$3)
G506:  @INT((G425/$A$434)*$D$3)
H506:  @INT((H425/$A$434)*$D$3)
B507:  @INT((B426/$A$434)*$D$3)
C507:  @INT((C426/$A$434)*$D$3)
D507:  @INT((D426/$A$434)*$D$3)
E507:  @INT((E426/$A$434)*$D$3)
F507:  @INT((F426/$A$434)*$D$3)
G507:  @INT((G426/$A$434)*$D$3)
H507:  @INT((H426/$A$434)*$D$3)
B508:  @INT((B427/$A$434)*$D$3)
C508:  @INT((C427/$A$434)*$D$3)
E508:  @INT((E427/$A$434)*$D$3)
```

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```
F508:  @INT((F427/SAS4)=SDS3)
G508:  @INT((G427/SAS4S4)=SDS3)
H508:  @INT((H427/SAS4S4)=SDS3)
B509:  @INT((B428/SAS4S4)=SDS3)
C509:  @INT((C428/SAS4. ()=SDS3)
D509:  @INT((D428/SAS4S4)=SDS3)
E509:  @INT((E428/SAS4S4)=SDS3)
F509:  @INT((F428/SAS4S4)=SDS3)
G509:  @INT((G428/SAS4S4)=SDS3)
H509:  @INT((H428/SAS4S4)=SDS3)
```

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Appendix B: Attenuation Spreadsheet Formulas

This appendix contains a printout of all the formulas used in the attenuation spreadsheet. As in Appendix A, each cell of the spreadsheet is listed on a separate line and the cell labels are set up so that letters represent columns and numbers represent rows starting from the upper left corner of the spreadsheet. The symbols used in front of the cell entries are the same as in Appendix A also.

The first row contains variables used for the calculations in the rest of the spreadsheet. The variables are the time after dispensing and the cloud scaling size. The second row calculates the 'z/V' term from Eq (42).

Rows 4 through 70 of the spreadsheet are reserved for the values in rows 443 through 509 of the density spreadsheet. In practice, the entries are transferred by first copying the values from the density spreadsheet into a separate file and then transporting them to this spreadsheet. The cells in the printout contain only zeros since until the data transferral occurs, the cells in this spreadsheet are empty.

The remainder of the spreadsheet calculates values for signal attenuation through the central column of the chaff cloud. Each of the 49 subsections in this portion of the

spreadsheet calculates the decimal fraction of the incident

signal which penetrates through sections of the cloud

corresponding to those in Figure 8.

A1: "t =
B1: 3600
C1: 'sec
D1: 0.05
E1: '(Z ln Ke)
A2: 1/(Q1)^2
B2: '(Z/Vol)
D2: 0.01
E2: 'm/s
A4: 0
E4: 10
C4: 214
D4: 605
E4: 237
F4: 12
G4: 0
A5: 10
B5: 1479
C5: 30141
D5: 85178
E5: 33382
F5: 1814
G5: 13
A6: 195
B6: 28759
C6: 586030
D6: 1656092
E6: 649037
F6: 35275
G6: 265
A7: 526
B7: 77332
C7: 1575812
D7: 4453162
E7: 1745235
F7: 94855
G7: 714
A8: 195
B8: 28759
C8: 586030
D8: 1656092
E8: 649037
F8: 35275
G8: 265
A9: 10
B9: 1479
C9: 30141
D9: 85178
E9: 33382
F9: 1814
G9: 13
A10: 0
B10: 10

C10: 214
D10: 605
E10: 237
F10: 12
G10: 0
A14: 7
B14: 1063
C14: 21298
D14: 59172
E14: 22798
F14: 1218
G14: 9
A15: 1034
B15: 149456
C15: 2994101
D15: 8318384
E15: 3205035
F15: 171256
G15: 1269
A16: 20115
B16: 2905812
C16: 58212812
D16: 161730160
E16: 62313894
F16: 3329662
G16: 24673
A17: 34090
B17: 7813607
C17: 156531801
D17: 434885594
E17: 167559439
F17: 8953322
G17: 66346
A18: 20115
B18: 2905812
C16: 58212812
D18: 161730160
E18: 62313894
F18: 3329662
G18: 24673
A19: 1034
B19: 149456
C19: 2994101
D19: 8318384
E19: 3205035
F19: 171256
G19: 1269
A20: 7
B20: 1063
C20: 21298
D20: 59172
E20: 22798
F20 1218

G20: 9
A24: 121
B24: 17187
C24: 338502
D24: 924576
E24: 350223
F24: 18397
G24: 134
A25: 17013
B25: 2416142
C25: 47586309
D25: 129975915
E25: 49234007
F25: 2586359
G25: 18842
A26: 330778
B26: 46975847
C26: 925198524
D26: 2527056187
E26: 957231974
F26: 50285287
G26: 366341
A27: 885448
B27: 126316075
C27: 2467814782
D27: 67951477 4
E27: 2573956489
F27: 135215021
G27: 985076
A28: 330778
B28: 46975847
C28: 925106524
D28: 2527056167
E28: 957231974
F28: 50285287
G28: 366341
A29: 17013
B29: 2416142
C29: 47586309
D29: 129975915
E29: 49234007
F29: 2586359
G29: 18842
A30: 121
B30: 17187
C30: 338502
D30: 924576
E30: 350223
F30: 18397
G30: 134
A31: 318
B34: 44455
C31: 860785

D34: 2311448
E34: 860785
F34: 44455
G34: 318
A35: 44761
B35: 6249525
C35: 121008227
D35: 324940750
E35: 121008227
F35: 6249525
G35: 44761
A36: 870267
B36: 121506377
C36: 2352701946
D36: 6317659159
E36: 2352701946
F36: 121506377
G36: 870267
A37: 2340112
B37: 326725533
F37. 6326316511
D37: 16987919575
E37: 6326316511
F37: 326725533
G37: 2340112
A38: 870267
B38: 121506377
C38: 2352701946
D38: 6317659159
E38: 2352701946
F38: 121506377
G38: 870267
A39: 44761
B39: 6249525
C39: 121008227
D39: 324940750
E39: 121008227
F39: 6249525
G39: 44761
A40: 318
B40: 44455
C40: 860785
D40: 2311448
E40: 860785
F40: 44455
G40: 318
A44: 134
B44: 18397
C44: 350223
D44: 924576
E44: 338502
F44: 17187
G44: 121

A45: 18842
B45: 2586359
C45: 49234007
D45: 129875915
E45: 47586309
F45: 2416142
G45: 17013
A46: 366341
B46: 50285287
C46: 957231974
D46: 2527058187
E46: 925196824
F46: 46975847
G46: 330778
A47: 985076
B47: 135215021
C47: 2573956489
D47: 6725147724
E47: 2487814782
F47: 126316075
G47: 889448
A48: 366341
B48: 50285287
C48: 957231374
D48: 2527056187
E48: 925196824
F48: 46975847
G48: 330778
A49: 18842
B49: 2586359
C49: 49234007
D49: 129875915
E49: 47586309
F49: 2416142
G49: 17013
A50: 134
B50: 18397
C50: 350223
D50: 924576
E50: 330502
F50: 17187
G50: 121
A54: 0
B54: 1218
C54: 22798
D54: 59172
E54: 21298
F54: 1063
G54: 7
A55: 1269
B55: 171256
C55: 3205035
D55: 8318384

E55: 2994101
F55: 149456
G55: 1034
A56: 24673
B56: 3329662
C56: 62313694
D56: 161730160
E56: 58212812
F56: 2905812
G56: 20115
A57: 66346
B57: 8953322
C57: 167559435
D57: 434885594
E57: 156531601
F57: 7813607
G57: 54090
A58: 24673
B58: 3329662
C58: 62313894
D58: 161730160
E58: 58212812
F58: 2905812
G58: 20115
A59: 1269
B59: 171256
C59: 3205035
D59: 8318384
E59: 2994101
F59: 149456
G59: 1034
A60: 9
B60: 1218
C60: 22798
D60: 59172
E60: 21298
F60: 1063
G60: 7
A64: 0
B64: 12
C64: 237
D64: 605
E64: 214
F64: 10
G64: 0
A65: 13
B65: 1814
C65: 33382
D65: 85178
E65: 30141
F65: 1479
G65: 10
A66: 265

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B66:  35275
C66:  649037
D66:  1656092
E66:  586030
F66:  28759
G66:  195
A67:  714
B67:  94855
C67:  1745235
D67:  4453162
E67:  1575812
F67:  77332
G67:  526
A68:  265
B68:  35275
C68:  649037
D68:  1656092
E68:  586030
F68:  28759
G68:  195
A69:  13
B69:  1814
C69:  33382
D69:  85178
E69:  30141
F69:  1479
G69:  10
A70:  0
B70:  12
C70:  237
D70:  605
E70:  214
F70:  10
G70:  0
A72:  '-
B72:  'CALCULATIONS
A74:  'Pout(1,1)
B74:  @EXP(-$A4*2.25E-10*$A$2)
C74:  (@EXP(-$A14*2.25E-10*$A$2))*B74
D74:  (@EXP(-$A24*2.25E-10*$A$2))*C74
E74:  (@EXP(-$A34*2.25E-10*$A$2))*D74
F74:  (@EXP(-$A44*2.25E-10*$A$2))*E74
G74:  (@EXP(-$A54*2.25E-10*$A$2))*F74
H74:  (@EXP(-$A64*2.25E-10*$A$2))*G74
A75:  'Pout(1,2)
B75:  @EXP(-$A5*2.25E-10*$A$2)
C75:  (@EXP(-$A15*2.25E-10*$A$2))*B75
D75:  (@EXP(-$A25*2.25E-10*$A$2))*C75
E75:  (@EXP(-$A35*2.25E-10*$A$2))*D75
F75:  (@EXP(-$A45*2.25E-10*$A$2))*E75
G75:  (@EXP(-$A55*2.25E-10*$A$2))*F75
H75:  (@EXP(-$A65*2.25E-10*$A$2))*G75
A76:  'Pout(1,3)

B76:  (@EXP(-$A6*2.25E-10*$A$2)
C76:  (@EXP(-$A16*2.25E-10*$A$2))*B76
D76:  (@EXP(-$A26*2.25E-10*$A$2))*C76
E76:  (@EXP(-$A36*2.25E-10*$A$2))*D76
F76:  (@EXP(-$A46*2.25E-10*$A$2))*E76
G76:  (@EXP(-$A56*2.25E-10*$A$2))*F76
H76:  (@EXP(-$A66*2.25E-10*$A$2))*G76
A77:  'Pout(1,4)
B77:  (@EXP(-$A7*2.25E-10*$A$2)
C77:  (@EXP(-$A17*2.25E-10*$A$2))*B77
D77:  (@EXP(-$A27*2.25E-10*$A$2))*C77
E77:  (@EXP(-$A37*2.25E-10*$A$2))*D77
F77:  (@EXP(-$A47*2.25E-10*$A$2))*E77
G77:  (@EXP(-$A57*2.25E-10*$A$2))*F77
H77:  (@EXP(-$A67*2.25E-10*$A$2))*G77
A78:  'Pout(1,5)
B78:  @EXP(-$A8*2.25E-10*$A$2)
C78:  (@EXP(-$A18*2.25E-10*$A$2))*B78
D78:  (@EXP(-$A28*2.25E-10*$A$2))*C78
E78:  (@EXP(-$A38*2.25E-10*$A$2))*D78
F78:  (@EXP(-$A48*2.25E-10*$A$2))*E78
G78:  (@EXP(-$A58*2.25E-10*$A$2))*F78
H78:  (@EXP(-$A68*2.25E-10*$A$2))*G78
A79:  'Pout(1,6)
B79:  @EXP(-$A9*2.25E-10*$A$2)
C79:  (@EXP(-$A19*2.25E-10*$A$2))*B79
D79:  (@EXP(-$A29*2.25E-10*$A$2))*C79
E79:  (@EXP(-$A39*2.25E-10*$A$2))*D79
F79:  (@EXP(-$A49*2.25E-10*$A$2))*E79
G79.  (@EXP(-$A59*2.25E-10*$A$2))*F79
H79:  (@EXP(-$A69*2.25E-10*$A$2))*G79
A80:  'Pout(1,7)
B80:  @EXP(-$A10*2.25E-10*$A$2)
C80:  (@EXP(-$A20*2.25E-10*$A$2))*B80
D80:  (@EXP(-$A30*2.25E-10*$A$2))*C80
E30:  (@EXP(-$A40*2.25E-10*$A$2))*D80
F80:  (@EXP(-$A50*2.25E-10*$A$2))*E80
H80:  (@EXP(-$A70*2.25E-10*$A$2))*G80
A82:  'Pout(2,1)
B82:  @EXP(-$B4*2.25E-10*$A$2)
C82:  (@EXP(-$B14*2.25E-10*$A$2))*B82
D82:  (@EXP(-$B24*2.25E-10*$A$2))*C82
E82:  (@EXP(-$B34*2.25E-10*$A$2))*D82
F82:  (@EXP(-$B44*2.25E-10*$A$2))*E82
G82:  (@EXP(-$B54*2.25E-10*$A$2))*F82
H82:  (@EXP(-$B64*2.25E-10*$A$2))*G82
A83:  'Pout(2,2)
B83:  @EXP(-$B5*2.25E-10*$A$2)
C83:  (@EXP(-$B15*2.25E-10*$A$2))*B83
D83:  (@EXP(-$B25*2.25E-10*$A$2))*C83
E83:  (@EXP(-$B35*2.25E-10*$A$2))*D83
F83:  (@EXP(-$B45*2.25E-10*$A$2))*E83
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G83:
H83:  (@EXP(-$B55*2.25E-10*$A$2))*F83
A84:  'Pout(2,3)
B84:  @EXP(-$B6*2.25E-10*$A$2)
C84:  (@EXP(-$B16*2.25E-10*$A$2))*B84
D84:  (@EXP(-$B26*2.25E-10*$A$2))*C84
E84:  (@EXP(-$B36*2.25E-10*$A$2))*D84
F84:  (@EXP(-$B46*2.25E-10*$A$2))*E84
G84:  (@EXP(-$B56*2.25E-10*$A$2))*F84
H84:  (@EXP(-$B66*2.25E-10*$A$2))*G84
A85:  'Pout(2,4)
B85:  @EXP(-$B7*2.25E-10*$A$2)
C85:  (@EXP(-$B17*2.25E-10*$A$2))*B85
D85:  (@EXP(-$B27*2.25E-10*$A$2))*C85
E85:  (@EXP(-$B37*2.25E-10*$A$2))*D85
F85:  (@EXP(-$B47*2.25E-10*$A$2))*E85
G85:  (@EXP(-$B57*2.25E-10*$A$2))*F85
H85:  (@EXP(-$B67*2.25E-10*$A$2))*G85
A86:  'Pout(2,5)
B86:  @EXP(-$B8*2.25E-10*$A$2)
C86:  (@EXP(-$B18*2.25E-10*$A$2))*B86
D86:  (@EXP(-$B28*2.25E-10*$A$2))*C86
E86:  (@EXP(-$B38*2.25E-10*$A$2))*D86
F86:  (@EXP(-$B48*2.25E-10*$A$2))*E86
G86:  (@EXP(-$B58*2.25E-10*$A$2))*F86
H86:  (@EXP(-$B68*2.25E-10*$A$2))*G86
A87:  'Pout(2,6)
B87:  @EXP(-$B9*2.25E-10*$A$2)
C87:  (@EXP(-$B19*2.25E-10*$A$2))*B87
D87:  (@EXP(-$B29*2.25E-10*$A$2))*C87
E87:  (@EXP(-$B39*2.25E-10*$A$2))*D87
F87:  (@EXP(-$B49*2.25E-10*$A$2))*E87
G87:  (@EXP(-$B59*2.25E-10*$A$2))*F87
H87:  (@EXP(-$B69*2.25E-10*$A$2))*G87
A88:  'Pout(2,7)
B88:  @EXP(-$B10*2.25E-10*$A$2)
C88:  (@EXP(-$B20*2.25E-10*$A$2))*B88
D88:  (@EXP(-$B30*2.25E-10*$A$2))*C88
E88:  (@EXP(-$B40*2.25E-10*$A$2))*D88
F88:  (@EXP(-$B50*2.25E-10*$A$2))*E88
G88:  (@EXP(-$B60*2.25E-10*$A$2))*F88
H88:  (@EXP(-$B70*2.25E-10*$A$2))*G88
A90:  'Pout(3,1)
B90:  @EXP(-$C4*2.25E-10*$A$2)
C90:  (@EXP(-$C14*2.25E-10*$A$2))*B90
D90:  (@EXP(-$C24*2.25E-10*$A$2))*C90
E90:  (@EXP(-$C34*2.25E-10*$A$2))*D90
F90:  (@EXP(-$C44*2.25E-10*$A$2))*E90
G90:  (@EXP(-$C54*2.25E-10*$A$2))*F90
H90:  (@EXP(-$C64*2.25E-10*$A$2))*G90
A91:  'Pout(3,2)
B91:  @EXP(-$C5*2.25E-10*$A$2)
C91:  (@EXP(-$C15*2.25E-10*$A$2))*B91

D91:  (@EXP(-$C25*2.25E-10*$A$2))*C91
E91:  (@EXP(-$C35*2.25E-10*$A$2))*D91
F91:  (@EXP(-$C45*2.25E-10*$A$2))*E91
G91:  (@EXP(-$C55*2.25E-10*$A$2))*F91
H91:  (@EXP(-$C65*2.25E-10*$A$2))*G91
A92:  'Pout(3,3)
B92:  @EXP(-$C6*2.25E-10*$A$2)
C92:  (@EXP(-$C16*2.25E-10*$A$2))*B92
D92:  (@EXP(-$C26*2.25E-10*$A$2))*C92
E92:  (@EXP(-$C36*2.25E-10*$A$2))*D92
F92:  (@EXP(-$C46*2.25E-10*$A$2))*E92
G92:  (@EXP(-$C56*2.25E-10*$A$2))*F92
H92:  (@EXP(-$C66*2.25E-10*$A$2))*G92
A93:  'Pout(3,4)
B93:  @EXP(-$C7*2.25E-10*$A$2)
C93:  (@EXP(-$C17*2.25E-10*$A$2))*B93
D93:  (@EXP(-$C27*2.25E-10*$A$2))*C93
E93:  (@EXP(-$C37*2.25E-10*$A$2))*D93
F93:  (@EXP(-$C47*2.25E-10*$A$2))*E93
G93:  (@EXP(-$C57*2.25E-10*$A$2))*F93
H93:  (@EXP(-$C67*2.25E-10*$A$2))*G93
A94:  'Pout(3,5)
B94:  @EXP(-$C8*2.25E-10*$A$2)
C94:  (@EXP(-$C18*2.25E-10*$A$2))*B94
D94:  (@EXP(-$C28*2.25E-10*$A$2))*C94
E94:  (@EXP(-$C38*2.25E-10*$A$2))*D94
F94:  (@EXP(-$C48*2.25E-10*$A$2))*E94
G94:  (@EXP(-$C58*2.25E-10*$A$2))*F94
H94:  (@EXP(-$C68*2.25E-10*$A$2))*G94
A95:  'Pout(3,6)
B95:  @EXP(-$C9*2.25E-10*$A$2)
C95:  (@EXP(-$C19*2.25E-10*$A$2))*B95
D95:  (@EXP(-$C29*2.25E-10*$A$2))*C95
E95:  (@EXP(-$C39*2.25E-10*$A$2))*D95
F95:  (@EXP(-$C49*2.25E-10*$A$2))*E95
G95:  (@EXP(-$C59*2.25E-10*$A$2))*F95
H95:  (@EXP(-$C69*2.25E-10*$A$2))*G95
A96:  'Pout(3,7)
B96:  @EXP(-$C10*2.25E-10*$A$2)
C96:  (@EXP(-$C20*2.25E-10*$A$2))*B96
D96:  (@EXP(-$C30*2.25E-10*$A$2))*C96
E96:  (@EXP(-$C40*2.25E-10*$A$2))*D96
F96:  (@EXP(-$C50*2.25E-10*$A$2))*E96
G96:  (@EXP(-$C60*2.25E-10*$A$2))*F96
H96:  (@EXP(-$C70*2.25E-10*$A$2))*G96
A98:  'Pout(4,1)
B98:  @EXP(-$D4*2.25E-10*$A$2)
C98:  (@EXP(-$D14*2.25E-10*$A$2))*B98
D98:  (@EXP(-$D24*2.25E-10*$A$2))*C98
E98:  (@EXP(-$D34*2.25E-10*$A$2))*D98
F98:  (@EXP(-$D44*2.25E-10*$A$2))*E98
G98:  (@EXP(-$D54*2.25E-10*$A$2))*F98
H98:  (@EXP(-$D64*2.25E-10*$A$2))*G98
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A99:   'Pout(4,2)
B99:   @EXP(-$D5*2.25E-10*$A$2)
C99:   (@EXP(-$D15*2.25E-10*$A$2))*B99
D99:   (@EXP(-$D25*2.25E-10*$A$2))*C99
E99:   (@EXP(-$D35*2.25E-10*$A$2))*D99
F99:   (@EXP(-$D45*2.25E-10*$A$2))*E99
G99:   (@EXP(-$D55*2.25E-10*$A$2))*F99
H99:   (@EXP(-$D65*2.25E-10*$A$2))*G99
A100:  'Pout(,3)
B100:  @EXP(-$D6*2.25E-10*$A$2)
C100:  (@EXP(-$D16*2.25E-10*$A$2))*B100
D100:  (@EXP(-$D26*2.25E-10*$A$2))*C100
E100:  (@EXP(-$D36*2.25E-10*$A$2))*D100
F100:  (@EXP(-$D46*2.25E-10*$A$2))*E100
G100:  (@EXP(-$D56*2.25E-10*$A$2))*F100
H100:  (@EXP(-$D66*2.25E-10*$A$2))*G100
A101:  'Pout(4,4)
B101:  @EXP(-$D7*2.25E-10*$A$2)
C101:  (@EXP(-$D17*2.25E-10*$A$2))*B101
D101:  (@EXP(-$D27*2.25E-10*$A$2))*C101
E101:  (@EXP(-$D37*2.25E-10*$A$2))*D101
F101:  (@EXP(-$D47*2.25E-10*$A$2))*E101
G101:  (@EXP(-$D57*2.25E-10*$A$2))*F101
H101:  (@EXP(-$D67*2.25E-10*$A$2))*G101
A102:  'Pout(4,5)
B102:  @EXP(-$D8*2.25E-10*$A$2)
C102:  (@EXP(-$D18*2.25E-10*$A$2))*B102
D102:  (@EXP(-$D28*2.25E-10*$A$2))*C102
E102:  (@EXP(-$D38*2.25E-10*$A$2))*D102
F102:  (@EXP(-$D48*2.25E-10*$A$2))*E102
G102:  (@EXP(-$D58*2.25E-10*$A$2))*F102
H102:  (@EXP(-$D68*2.25E-10*$A$2))*G102
A103:  'Pout(4,F)
B103:  @EXP(-$D9*2.25E-10*$A$2)
C103:  (@EXP(-$D19*2.25E-10*$A$2))*B103
D103:  (@EXP(-$D29*2.25E-10*$A$2))*C103
E103:  (@EXP(-$D39*2.25E-10*$A$2))*E103
F103:  (@EXP(-$D49*2.25E-10*$A$2))*F103
G103:  (@EXP(-$D59*2.25E-10*$A$2))*G103
H103:  (@EXP(-$D69*2.25E-10*$A$2))*G102
A104:  'Pout(4,7)
B104:  @EXP(-$D10*2.25E-10*$A$2)
C104:  (@EXP(-$D20*2.25E-10*$A$2))*B104
D104:  (@EXP(-$D30*2.25E-10*$A$2))*C104
E104:  (@EXP(-$D40*2.25E-10*$A$2))*E104
F104:  (@EXP(-$D50*2.25E-10*$A$2))*F104
G104:  (@EXP(-$D60*2.25E-10*$A$2))*G104
H104:  (@EXP(-$D70*2.25E-10*$A$2))*G104
A106:  'Pout(5,1)
B106:  @EXP(-$E4*2.25E-10*$A$2)
C106:  (@EXP(-$E14*2.25E-10*$A$2))*B106
D106:  (@EXP(-$E24*2.25E-10*$A$2))*C106
E106:  (@EXP(-$E34*2.25E-10*$A$2))*D106

F106:  (@EXP(-$E44*2.25E-10*$A$2))*E106
G106:  (@EXP(-$E54*2.25E-10*$A$2))*F106
H106:  (@EXP(-$E64*2.25E-10*$A$2))*G106
A107.  'Pout(5,2)
B107:  @EXP(-$E5*2.25E-10*$A$2)
C107:  (@EXP(-$E15*2.25E-10*$A$2))*B107
D107:  (@EXP(-$E25*2.25E-10*$A$2))*C107
E107:  (@EXP(-$E35*2.25E-10*$A$2))*D107
F107:  (@EXP(-$E45*2.25E-10*$A$2))*E107
G107:  (@EXP(-$E55*2.25E-10*$A$2))*F107
H107:  (@EXP(-$E65*2.25E-10*$A$2))*G107
A108:  'Pout(5,3)
B108:  @EXP(-$E6*2.25E-10*$A$2)
C108:  (@EXP(-$E16*2.25E-10*$A$2))*B108
D108:  (@EXP(-$E26*2.25E-10*$A$2))*C108
E108:  (@EXP(-$E36*2.25E-10*$A$2))*D108
F108:  (@EXP(-$E46*2.25E-10*$A$2))*E108
G108:  (@EXP(-$E56*2.25E-10*$A$2))*F108
H108:  (@EXP(-$E66*2.25E-10*$A$2))*G108
A109:  'Pout(5,4)
B109:  @EXP(-$E7*2.25E-10*$A$2)
C109:  (@EXP(-$E17*2.25E-10*$A$2))*B109
D109:  (@EXP(-$E27*2.25E-10*$A$2))*C109
E109:  (@EXP(-$E37*2.25E-10*$A$2))*D109
F109:  (@EXP(-$E47*2.25E-10*$A$2))*E109
G109:  (@EXP(-$E57*2.25E-10*$A$2))*F109
H109:  (@EXP(-$E67*2.25E-10*$A$2))*G109
A110:  'Pout(5,5)
B110:  @EXP(-$E8*2.25E-10*$A$2)
C110:  (@EXP(-$E18*2.25E-10*$A$2))*B110
D110:  (@EXP(-$E28*2.25E-10*$A$2))*C110
E110:  (@EXP(-$E38*2.25E-10*$A$2))*D110
F110:  (@EXP(-$E48*2.25E-10*$A$2))*E110
G110:  (@EXP(-$E58*2.25E-10*$A$2))*F110
H110:  (@EXP(-$E68*2.25E-10*$A$2))*G110
A111:  'Pout(5,6)
B111:  @EXP(-$E9*2.25E-10*$A$2)
C111:  (@EXP(-$E19*2.25E-10*$A$2))*B111
D111:  (@EXP(-$E29*2.25E-10*$A$2))*C111
E111:  (@EXP(-$E39*2.25E-10*$A$2))*D111
F111:  (@EXP(-$E49*2.25E-10*$A$2))*E111
G111:  (@EXP(-$E59*2.25E-10*$A$2))*F111
H111:  (@EXP(-$E69*2.25E-10*$A$2))*G111
A112:  'Pout(5,7)
B112:  @EXP(-$E10*2.25E-10*$A$2)
C112:  (@EXP(-$E20*2.25E-10*$A$2))*B112
D112:  (@EXP(-$E30*2.25E-10*$A$2))*C112
E112:  (@EXP(-$E40*2.25E-10*$A$2))*D112
F112:  (@EXP(-$E50*2.25E-10*$A$2))*E112
G112:  (@EXP(-$E60*2.25E-10*$A$2))*F112
H112:  (@EXP(-$E70*2.25E-10*$A$2))*G112
A114:  'Pout(6,1)
B114:  @EXP(-$F4*2.25E-10*$A$2)
```

```
C114:  (@EXP(-$F14*2.25E-10*$A$2))*B114
D114:  (@EXP(-$F24*2.25E-10*$A$2))*C114
E114:  (@EXP(-$F34*2.25E-10*$A$2))*D114
F114:  (@EXP(-$F44*2.25E-10*$A$2))*E114
G114:  (@EXP(-$F54*2.25E-10*$A$2))*F114
H114:  (@EXP(-$F64*2.25E-10*$A$2))*G114
A115:  'Pout(6,2)
B115:  @EXP(-$F5*2.25E-10*$A$2)
C115:  (@EXP(-$F15*2.25E-10*$A$2))*B115
D115:  (@EXP(-$F25*2.25E-10*$A$2))*C115
E115:  (@EXP(-$F35*2.25E-10*$A$2))*D115
F115:  (@EXP(-$F45*2.25E-10*$A$2))*E115
G115:  (@EXP(-$F55*2.25E-10*$A$2))*F115
H115:  (@EXP(-$F65*2.25E-10*$A$2))*G115
A116:  'Pout(6,3)
B116:  @EXP(-$F6*2.25E-10*$A$2)
C116:  (@EXP(-$F16*2.25E-10*$A$2))*B116
D116:  (@EXP(-$F26*2.25E-10*$A$2))*C116
E116:  (@EXP(-$F36*2.25E-10*$A$2))*D116
F116:  (@EXP(-$F46*2.25E-10*$A$2))*E116
G116:  (@EXP(-$F56*2.25E-10*$A$2))*F116
H116:  (@EXP(-$F66*2.25E-10*$A$2))*G116
A117:  'Pout(6,4)
B117:  @EXP(-$F7*2.25E-10*$A$2)
C117:  (@EXP(-$F17*2.25E-10*$A$2))*B117
D117:  (@EXP(-$F27*2.25E-10*$A$2))*C117
E117:  (@EXP(-$F37*2.25E-10*$A$2))*D117
F117:  (@EXP(-$F47*2.25E-10*$A$2))*E117
G117:  (@EXP(-$F57*2.25E-10*$A$2))*F117
H117:  (@EXP(-$F67*2.25E-10*$A$2))*G117
A118:  'Pout(6,5)
B118:  @EXP(-$F8*2.25E-10*$A$2)
C118:  (@EXP(-$F18*2.25E-10*$A$2))*B118
D118:  (@EXP(-$F28*2.25E-10*$A$2))*C118
E118:  (@EXP(-$F38*2.25E-10*$A$2))*D118
F118:  (@EXP(-$F48*2.25E-10*$A$2))*E118
G118:  (@EXP(-$F58*2.25E-10*$A$2))*F118
H118:  (@EXP(-$F68*2.25E-10*$A$2))*G118
A119:  'Pout(6,6)
B119:  @EXP(-$F9*2.25E-10*$A$2)
C119:  (@EXP(-$F19*2.25E-10*$A$2))*B119
D119:  (@EXP(-$F29*2.25E-10*$A$2))*C119
E119:  (@EXP(-$F39*2.25E-10*$A$2))*D119
F119:  (@EXP(-$F49*2.25E-10*$A$2))*E119
G119:  (@EXP(-$F59*2.25E-10*$A$2))*F119
H119:  (@EXP(-$F69*2.25E-10*$A$2))*G119
A120:  'Pout(6,7)
B120:  @EXP(-$F10*2.25E-10*$A$2)
C120:  (@EXP(-$F20*2.25E-10*$A$2))*B120
D120:  (@EXP(-$F30*2.25E-10*$A$2))*C120
E120:  (@EXP(-$F40*2.25E-10*$A$2))*D120
F120:  (@EXP(-$F50*2.25E-10*$A$2))*E120
G120:  (@EXP(-$F60*2.25E-10*$A$2))*F120

H120:  (@EXP(-$F70*2.25E-10*$A$2))*G120
A122:  'Pout(7,1)
B122:  @EXP(-$G4*2.25E-10*$A$2)
C122:  (@EXP(-$G14*2.25E-10*$A$2))*B122
D122:  (@EXP(-$G24*2.25E-10*$A$2))*C122
E122:  (@EXP(-$G34*2.25E-10*$A$2))*D122
F122:  (@EXP(-$G44*2.25E-10*$A$2))*E122
G122:  (@EXP(-$G54*2.25E-10*$A$2))*F122
H122:  (@EXP(-$G64*2.25E-10*$A$2))*G122
A123:  'Pout(7,2)
B123:  @EXP(-$G5*2.25E-10*$A$2)
C123:  (@EXP(-$G15*2.25E-10*$A$2))*B123
D123:  (@EXP(-$G25*2.25E-10*$A$2))*C123
E123:  (@EXP(-$G35*2.25E-10*$A$2))*D123
F123:  (@EXP(-$G45*2.25E-10*$A$2))*E123
G123:  (@EXP(-$G55*2.25E-10*$A$2))*F123
H123:  (@EXP(-$G65*2.25E-10*$A$2))*G123
A124:  'Pout(7,3)
B124:  @EXP(-$G6*2.25E-10*$A$2)
C124:  (@EXP(-$G16*2.25E-10*$A$2))*B124
D124:  (@EXP(-$G26*2.25E-10*$A$2))*C124
E124:  (@EXP(-$G36*2.25E-10*$A$2))*D124
F124:  (@EXP(-$G46*2.25E-10*$A$2))*E124
G124:  (@EXP(-$G56*2.25E-10*$A$2))*F124
H124:  (@EXP(-$G66*2.25E-10*$A$2))*G124
A125:  'Pout(7,4)
B125:  @EXP(-$G7*2.25E-10*$A$2)
C125:  (@EXP(-$G17*2.25E-10*$A$2))*B125
D125:  (@EXP(-$G27*2.25E-10*$A$2))*C125
E125:  (@EXP(-$G37*2.25E-10*$A$2))*D125
F125:  (@EXP(-$G47*2.25E-10*$A$2))*E125
G125:  (@EXP(-$G57*2.25E-10*$A$2))*F125
H125:  (@EXP(-$G67*2.25E-10*$A$2))*G125
A126:  'Pout(7,5)
B126:  @EXP(-$G8*2.25E-10*$A$2)
C126:  (@EXP(-$G18*2.25E-10*$A$2))*B126
D126:  (@EXP(-$G28*2.25E-10*$A$2))*C126
E126:  (@EXP(-$G38*2.25E-10*$A$2))*D126
F126:  (@EXP(-$G48*2.25E-10*$A$2))*E126
G126:  (@EXP(-$G58*2.25E-10*$A$2))*F126
H126:  (@EXP(-$G68*2.25E-10*$A$2))*G126
A127:  'Pout(7,6)
B127:  @EXP(-$G9*2.25E-10*$A$2)
C127:  (@EXP(-$G19*2.25E-10*$A$2))*B127
D127:  (@EXP(-$G29*2.25E-10*$A$2))*C127
E127:  (@EXP(-$G39*2.25E-10*$A$2))*D127
F127:  (@EXP(-$G49*2.25E-10*$A$2))*E127
G127:  (@EXP(-$G59*2.25E-10*$A$2))*F127
H127:  (@EXP(-$G69*2.25E-10*$A$2))*G127
A128:  'Pout(7,7)
B128:  @EXP(-$G10*2.25E-10*$A$2)
C128:  (@EXP(-$G20*2.25E-10*$A$2))*B128
D128:  (@EXP(-$G30*2.25E-10*$A$2))*C128
```

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E128: (@EXP(-$G$40*2.25E-10*$A$2))*D128
F128: (@EXP(-$G$50*2.25E-10*$A$2))*E12
G128: (@EXP(-$G$60*2.25E-10*$A$2))*F128
H128: (@EXP(-$G$70*2.25E-10*$A$2))*G128
```

Appendix C: An Example Calculation

To help clarify the description provided in Chapter IV
of the procedures for determining signal attenuation, an
example describing the calculation of signal attenuation
acheived by the chaff cloud is provided below. For this
example, the dispensing velocity was 0.05 m/s and the time
after initial dispensing was six hours.

First, the cloud model using the density spreadsheet
was scaled up by factors of seven (for ease of subsequent
calculations, and rescaling) until all of the chaff
particles were contained within the subvolume at the origin
of the grid. The size of this cube (cube A) was 120.05 km
on a side with the subvolume (cube B) 17.15 km on a side.
This is shown in Figure C1. The grid could now be
downscaled by a factor of seven and still contain all the
chaff particles inside.

Next, the density spreadsheet was rerun for subvolumes
2.45 km on a side to determine the extent of the cloud in
the ξ direction within cube B. Using this procedure, the
cloud was found to extend into (but not completely through)
one subvolume on each side of the central subvolume of cube
B (cubes D and E). Thus, the cloud was only 7.35 km thick
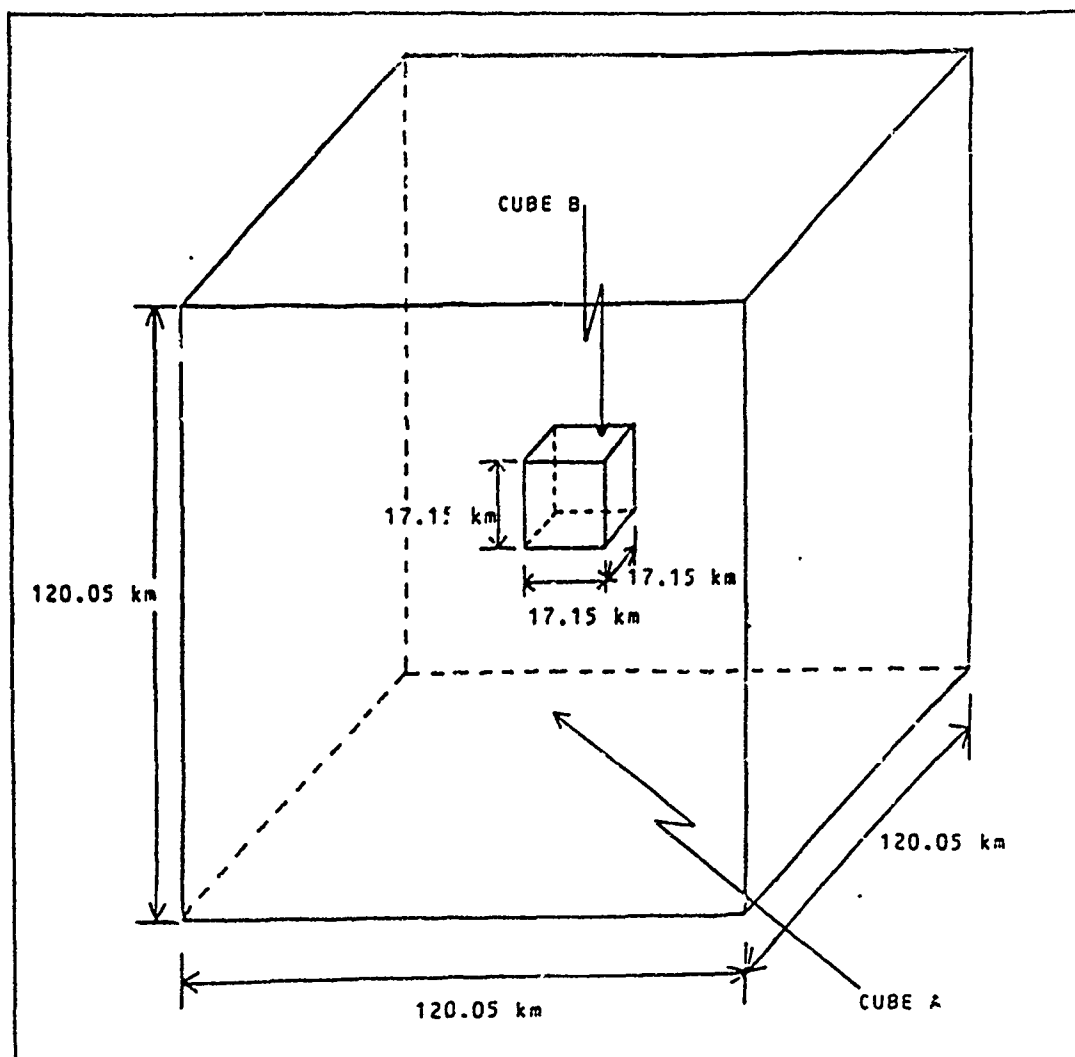in the ξ direction at this point in time. This is shown in

Figure C1.   Initial Downscaling of Cube A

Figure C2. The number of particles within each of these three subvolumes was then recorded for the next subscaling.

It is important to note here that only those subvolumes which contained a significant number of particles at each subscaling were considered further in the attenuation calculations. A significant number of particles was determined to be the number of particles which caused an attenuation of 0.1 percent or greater when Eq (42) was applied. For a subvolume of 0.35 km on a side, this number was determined to be approximately 545,000 particles. For a subvolume 2.45 km on a side, 26.6 million particles was considered a significant number.

Now, the spreadsheet was again used to downscale the cloud using dimensions of 0.35 km on a side. For the center subvolume from before (cube C), the origin was left at $(\xi, \eta, \zeta) = (0,0,0)$. However, for cubes D and E, the origin was moved 2.45 km (5.82×10^{-5} RU) in the plus and minus ξ directions, respectively. The number of chaff particles within the central column of cubes C, D, and E ((η, ζ) equal to (4,4)) for each of the seven values of ξ were now recorded. For this example, 21 sub-subvolumes 0.35 km on a side (cubes F through Z) were found to contain a significant number of particles to be recorded. Cubes F through Z are shown in Figure C3 and the number of chaff particles contained in each cube is shown in Table CI.
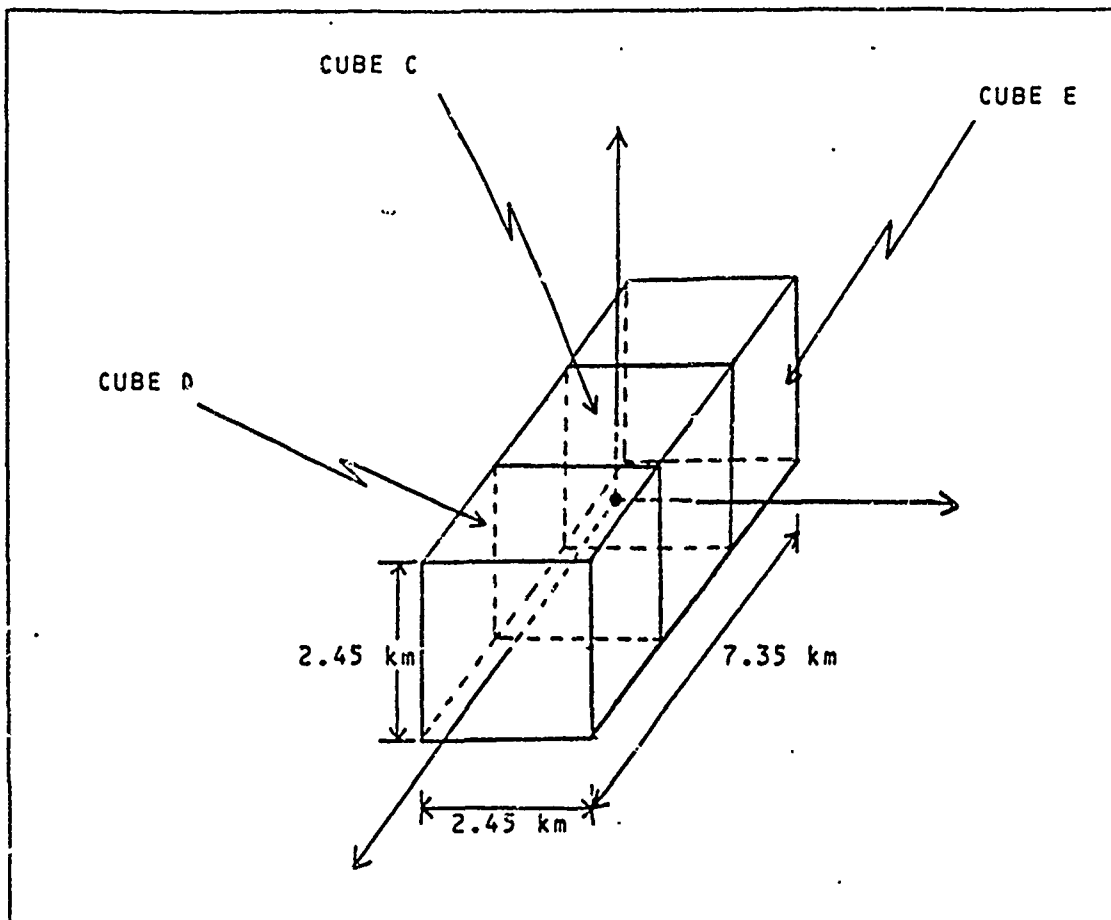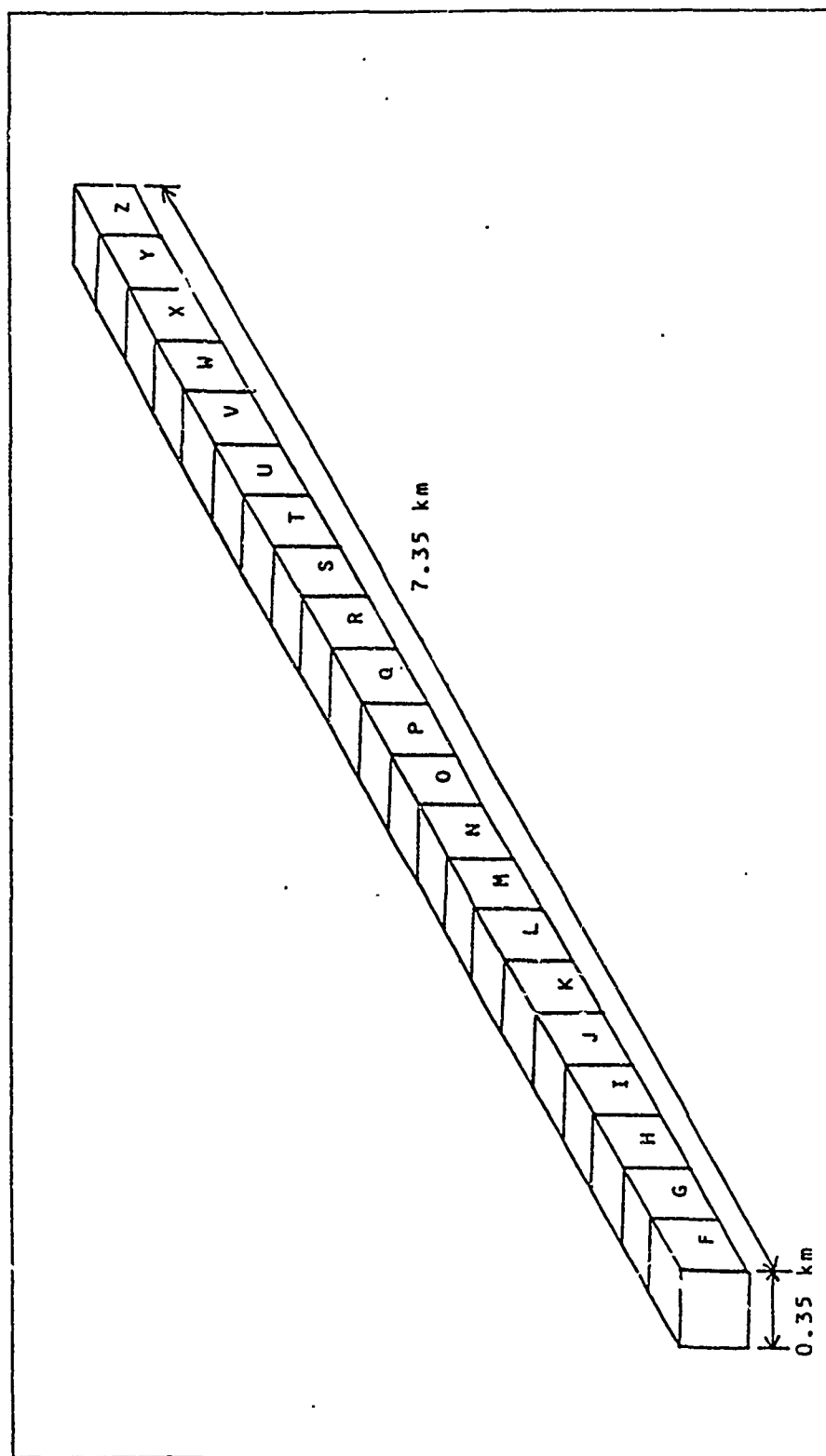
Figure C2. Central Column at 2.45 km Scaling

Figure C3. Central Column at 0.35 km Scaling

Table CI. Number of Particles Along the Central
Column at 0.35 km Scaling

CUBE	# OF PARTICLES
F	5.72×10^5
G	1.60×10^6
H	4.03×10^6
I	9.10×10^6
J	1.84×10^7
K	3.35×10^7
L	5.47×10^7
M	2.41×10^8
N	3.16×10^8
O	3.72×10^8
P	3.93×10^8
Q	3.72×10^8
R	3.16×10^8
S	2.41×10^8
T	5.42×10^7
U	3.32×10^7
V	1.83×10^7
W	9.02×10^6
X	4.00×10^6
Y	1.59×10^6
Z	5.67×10^5

Each of these 21 sub-subvolumes through the thickness
of the cloud was now examined individually. The scale of
the spreadsheet was again reduced to provide cubes 0.05 km
on a side. The origin of the grid was moved in each case to
the appropriate ξ location. Since only the central column
of the chaff cloud was examined in detail, the coordinates
for both η and ζ were always zero as the origin was moved
back and forth through the chaff cloud.

Finally, using the attenuation spreadsheet, the signal
attenuation through each of cubes F through Z was
calculated. Then, each of the 21 attenuation values for the
21 cubes with (η,ζ) equal to $(1,2)$ were multiplied together
to obtain the signal attenuation through the entire chaff
cloud at those coordinates. Attenuation for the other 44
significant (η,ζ) values were then calculated in a similar
manner.

Since the effectiveness of the chaff cloud is only as
good as the worst attenuation value, the highest value for
P_{out} obtained in the above calculations was chosen as the
overall attenuation value for the chaff cloud. Using the
above method, the signal attenuation for the chaff cloud
with a 0.05 m/s dispensing velocity and at a time six hours
after dispensing turned out to be -18.3 dB (P_{out}/P_{in} =
0.0149). Therefore, the cloud is "effective" for these
parameters of velocity and time after dispensing.

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The above procedure was repeated for every combination of dispensing velocity and time after dispensing that was examined in this study. Although the signal attenuation and the number of times the cloud was scaled up and down varied for each change in dispensing velocity and time after dispensing, the overall procedure was very similar for the additional runs required.

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release;
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GSO/ENP/88D-6	5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION School of Engineering	6b. OFFICE SYMBOL (If applicable) AFIT/ENS	7a. NAME OF MONITORING ORGANIZATION

6c. ADDRESS (City, State, and ZIP Code) Air Force Institute of Technology (AU) Wright-Patterson AFB, Ohio 45433-6583	7b. ADDRESS (City, State, and ZIP Code)

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER

8c. ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS			
	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
THE USE OF CHAFF IN SPACE AS A JAMMING DEVICE BETWEEN GROUND STATIONS AND SATELLITES

12. PERSONAL AUTHOR(S)
Alan R. Sterns, Capt USAF

13a. TYPE OF REPORT MS Thesis	13b. TIME COVERED FROM _____ TO _____	14 DATE OF REPORT (Year, Month, Day) 1988, DEC	15. PAGE COUNT 171

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Chaff, Statistical Mechanics, Orbits,
17	04	03	Spacecraft Debris, Electronic Countermeasures
22	01		

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

LtCol Howard E. Evans II

Approved for release in
accordance with AFR 190-1
[signature]
12 Jan 1989

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL LtCol Howard Evans, Physics Instructor	22b. TELEPHONE (Include Area Code) (513) 255-2012	22c. OFFICE SYMBOL AFIT/ENP

DD Form 1473, JUN 86   Previous editions are obsolete.   SECURITY CLASSIFICATION OF THIS PAGE

This study predicts the time evolution of the attenuation characteristics of a cnaff cloud deployed in orbit around the earth. The study consists of three parts: applying the statistical mechanics solution of a satellite breakup model by William Heard of the Naval Research Laboratory, solving for particle density at any time after dispensing, and calculating the attenuation of an 8 GHz signal through the cloud. The study shows that significant levels of signal attenuation can be achieved, with attenuations of greater than -50 dB lasting for several hours.